

## AN EMPIRICAL ‘LOWER BOUND’ ON FREE-SHEAR-FLOW NOISE

Jonathan B. Freund, Mingjun Wei

*University of Illinois at Urbana-Champaign, Department of Theoretical and Applied Mechanics,  
Urbana, IL 61801, USA*

*Summary* The adjoint of the linearized compressible flow equations is formulated in such a way that it can be used to optimize local control actuation to reduce the noise radiated by a free shear flow. Both the flow and the adjoint are solved numerically and without modeling approximations. Jet noise is the technological application of interest, and a two-dimensional mixing layer is studied as a model of the near-nozzle region of a jet. It is found that the correct small perturbations near the inflow can dramatically reduce the radiated noise. This approach allows us to establish an empirical lower bound on the noise radiation by a this flow subject to our constraints. More importantly, it gives us an opportunity to study its mechanisms by providing noisy and quieted versions of the same flow. Superficially, the unsteady flow dynamics are remarkably unchanged despite a 10dB reduction in the noise.

### BACKGROUND

Since the beginning of the jet age, the most significant noise reductions derive from lower nozzle velocities  $U$  and Lighthill’s famous acoustic power  $\mathcal{P} \propto U^8$  scaling,<sup>1</sup> but at fixed flow conditions it is also now being found that certain nozzle modifications can reduce noise with acceptable thrust losses. Unfortunately, there is no firm theory to guide their design and no known upper limit on their effectiveness. The greater flexibility afforded by active actuation at the nozzle holds even greater promise, but the complexity of the flow and the subtleness of the noise mechanisms have hampered modeling efforts seeking to guide its use. Turbulence models simply lack the fidelity necessary to provided the detailed theoretical noise sources derived in aeroacoustic theories. Our direct numerical simulation effort<sup>2</sup> has shown that despite accuracy concerns deriving from the large energy mismatch between the flow and its noise, the sound field of a turbulent jet flow can indeed be computed accurately, but these simulation are both expensive and have not yet yielded a simplifying principle that can be used to improve models. Large-eddy simulation holds long term promise for prediction, but its direct use in design optimization of complex geometries or of actuation is not imminent. It remains unclear how quite a jet can be made at fixed nominal jet exit conditions or if there is even any use in pursuing active control.

### APPROACH AND OBJECTIVES

In view of this we have developed an automatic procedure to optimize near-nozzle control actuation in a model jet to reduce its noise. It will be seen that the procedure is an iterative prediction-based optimization that employs a direct numerical simulation of the flow including its radiated sound coupled with a direct solution of the adjoint of the linearized flow equations. However, these requirements do not limit the usefulness of the approach for our objectives. We will use it to discover how quiet the unsteady structures in a free shear flow can be made, which is important from the perspective of assessing whether or not active control should be pursued at all. It will also provide us a unique opportunity to compare a loud and a slightly perturbed but significantly quieter flow. Ideally, it will eventually guide the development of general control laws.

The model jet flow we consider is the two-dimensional mixing layer shown schematically in figure 1. The unsteadiness in the mixing layer is randomly excited in an absorbing boundary zone extending upstream of the computational domain, through the subsequent control, of course, has no direct knowledge of this excitation. This two-dimensional model flow has several of the salient features of the near-nozzle flow of a jet yet can be simulated cheaply enough to allow us explore different parameters and types of controls. Still, tens of thousands of processor hours on parallel systems are necessary for the current study of various controls.

The flow equations are linearized about the time dependent flow solution and their adjoint is then formulated in such a way that the adjoint solution provides the sensitivity of the noise, defined in terms of acoustic pressure  $p'$  by

$$\mathcal{J} = \int_0^T \int_{\Omega} p' p' dx dt,$$

to changes in the control in  $\mathcal{C}$ .  $\Omega$  and  $\mathcal{C}$  are labeled in figure 1. The flow and adjoint solutions are both attained numerically by direct numerical solution. The mesh is fine enough to preclude the need for any artificial dissipation or other modeling approximations.

The sensitivity information provided by the adjoint solution is used in a conjugate-gradient minimization algorithm to find control  $\phi(\mathbf{x}, t)$  such that  $\mathcal{J}(\phi)$  is minimized. In this fundamental study, we consider the most general control possible. It appears as source terms in the compressible flow equations having compact support in  $\mathcal{C}$ . Each space-time point of the discrete representation of  $\phi(\mathbf{x}, t)$  is treated as an independent control parameter.

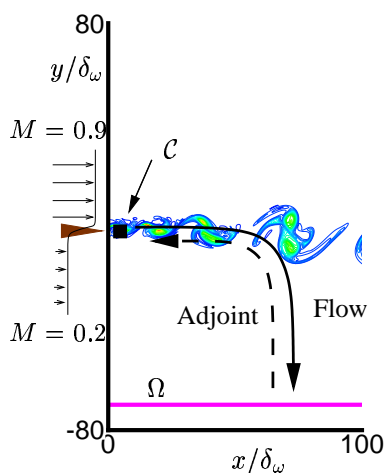
## RESULTS

After 10 to 20 line minimizations within the conjugate gradient search algorithm, the values of  $\mathcal{J}$  can be reduced by over a factor of 10 with minimal physical control effort. For example, when the control term is added to the  $y$ -momentum equation, which would crudely model the action of an actuator blowing normal to the shear layer, the peak instantaneous mechanical power is only 0.019% of the time averaged  $x$ -direction turbulence kinetic energy flux at  $\mathcal{C}$ . The details of the control identified are complex and are under further investigation. It is clear, however, that the action of  $\phi$  is convective, tracking flow structures as they pass.

The most notable result, perhaps, concerns how little the unsteady characteristics of the vortex roll-ups and pairings have changed. The net fluctuation kinetic energy at any particular downstream  $x$ -location is always within 14% of the uncontrolled case. Figure 2 visualizes the large structures in the mixing layer with vorticity before and after the control has been applied. There is clearly little difference. Subtracting controlled and uncontrolled fields suggest that the most significant change may be in the relative spacing of the eddies. Pairings and other such structural interactions are qualitatively unchanged by the control. The small difference may suggest that it is a simple noise cancellation (anti-sound) rather than a mechanistic change in the flow dynamics that reduces the radiated noise. However, a test simulation with the control region  $\mathcal{C}$  moved off the mixing layer only was able to reduce  $\mathcal{J}$  by less than a factor of 2, suggesting that it is indeed subtle changes in the flow dynamics that reduces the noise so significantly. Fourier transforming near-field flow quantities in the  $x$ -direction (giving wavenumbers  $k$ ) and time  $t$  (giving frequencies  $\omega$ ), shows that modes with radiation capable phase velocities ( $|\omega/k| > a_\infty$ ) experience a large (factor of 3 in cases) reduction after the control is optimized.

## CONCLUSIONS

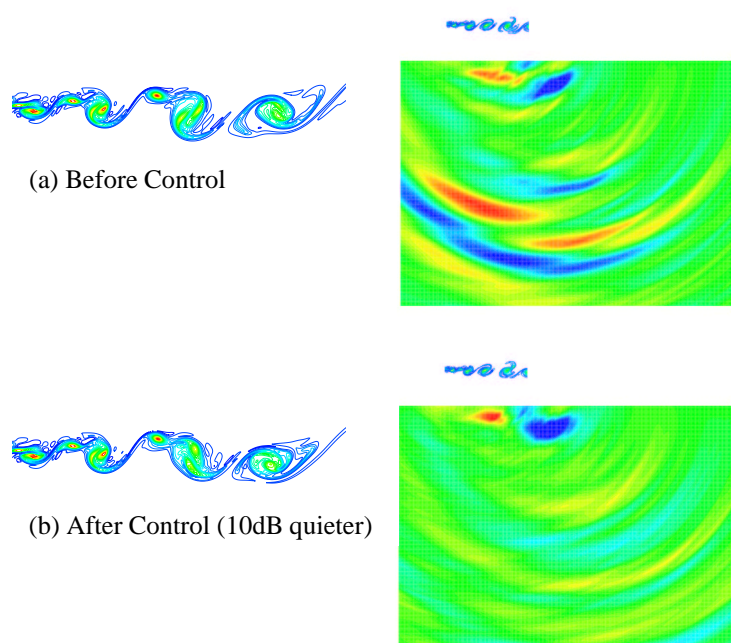
We have designed an automatic control optimization algorithm that substantially reduces the acoustic radiation of a two-dimensional mixing layer with minimal control effort, encouraging further pursuit of active control of free-shear-flow noise. The observable features of the flow are little changed by the control, suggesting that efforts to directly observe the noise making mechanics of jets are misguided. The only significant changes observed in our simulations thus far occur in the near-field and are in modes with radiation capable supersonic phase velocity. Physically, these are associated with the evolution of the unsteady flow structures as they advect downstream. (Funding from AFOSR is greatly appreciated.)



**Figure 1.** Schematic showing the target line for control  $\Omega$  and the region of support of the control  $\mathcal{C}$ . The arrows indicate the general direction of information transport: the numerical flow solution carries perturbations from the nozzle lip to the sound field and the adjoint solution carries the sensitivity of the noise from  $\Omega$  back to the region of control  $\mathcal{C}$ .

## References

- [1] M. J. Lighthill, "On sound generated aerodynamically: I. General theory," Proc. Royal Soc. Lond. A **211**, 564 (1952).
- [2] J. B. Freund, "Noise sources in a low-Reynolds-number turbulent jet at Mach 0.9," J. Fluid Mech. **438**, 277 (2001).



**Figure 2.** Vorticity magnitude visualization (contours) and pressure visualization (colours): (a) before adjoint-based optimization; (b) after adjoint-based optimization.