HYDRAULIC JUMPS AND RESONANCE IN GRAVITY-DRIVEN FLOWS OF LIQUIDS IN INCLINED WAVY CHANNELS: TRANSITION AND HYSTERESIS

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Summary We study the flow of a viscous liquid down an inclined channel with a sinusoidal bottom profile of moderate waviness. At low inclination angles, where basins form due to non-monotonous falling bottom slopes, we observe the formation of stationary hydraulic jumps in the form of shock fronts and surface rollers. There exists a bistable region in which both jump phenomena, shock fronts and surface rollers, can occur. At the low end of the bistable region, an instationary regime of a shock with a fingering-like lateral modulation is found. At higher volume flux and inclination angles, the hydraulic jump is suppressed by a standing wave that is generated by a resonance between gravity waves and the wavy bottom. At the transition between surface rollers and standing gravity-waves, periodic switching between the two occur.

INTRODUCTION

Sinusoidal bottom contours seem to be one of the simplest geometries to study the effect of bottom undulations on film and channel flows. The effects of the undulation on the flow increase with the waviness [1], [2] and also qualitatively new phenomena occur: For instance, a resonance of capillary waves with periodic bottom corrugations has been reported [3] and at high waviness, vortices are generated in the valleys of the bottom contour under creeping-flow conditions [4], [5]. For moderate waviness, gravity-driven flow leads to the creation of hydraulic jumps and standing waves once inertia becomes important. This is the region explored here.

EXPERIMENTAL SYSTEM AND SETUP

The experiments were carried out in a channel built from an aluminum bottom and transparent Plexiglas side-walls. It has a width of *170 mm* and is divided into a flat and an undulated section. The undulated part consists of three equal sinusoidal waves. The experiments were carried out around the central wave, which has a wavelength of *300 mm* and *15 mm* amplitude at inclination angles between 5° and 20°, thus in a range where the bottom contour has rising edges. The flat part serves to measure the film thickness. As liquids we chose a silicone oil with a kinematic viscosity *208 mm²/s* and a surface tension of *20.3 mN/m* at *299.15 K*. The surface contour were imaged directly with a CCD camera. For quantitative measurements of the surface contour, we used the PIV-system described in [3].

EXPERIMENTAL RESULTS

We have observed hydraulic jumps as shock fronts and surface rollers. Figure 1 depicts typical examples. Image (a) shows a shock front, which is straight except for bulges close to the wall, which are seen at the borders of the image, and in image (b) a surface roller is presented. At lower flow rate, the surface at the inflow into the basin, which is formed due to the rising edge of the bottom, the surface becomes undulated. With increasing flow rate the first undulation steepens and forms a shock front as shown in Figure 1(a). This shock front remains stable up to a certain flow rate. Beyond that the shock front is replaced by surface rollers. A further increase in the flow rate yields more pronounced surface rollers with surface profiles in flow directions that are more and more curved. Decreasing the flow rate, however, the surface rollers are maintained even below the critical flow rate for their formation, showing the

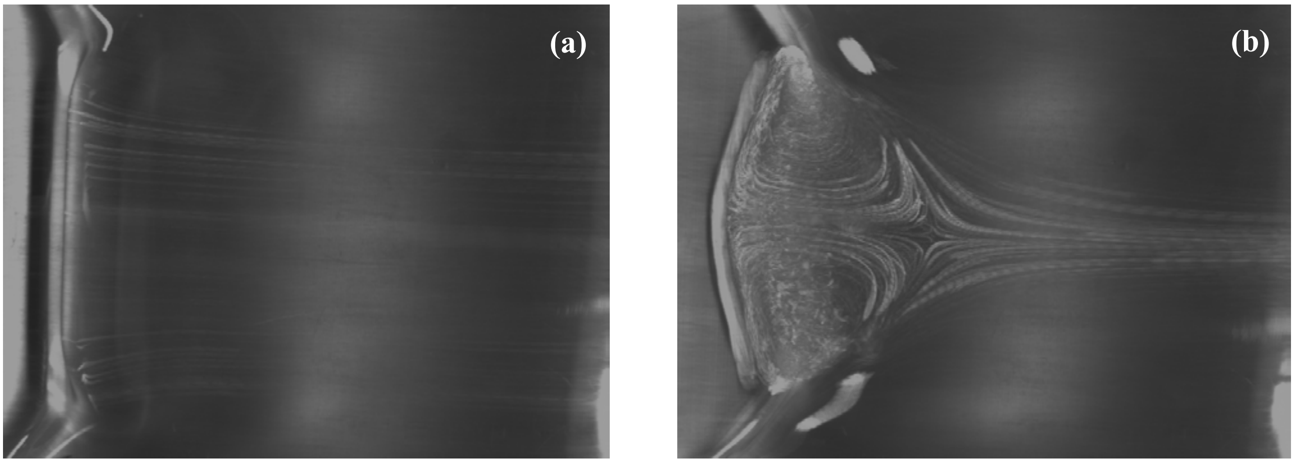

Figure 1: Top-view of the shock front (a) and the surface roller (b). The flow is from the left to the right. The flow is visualized with the air bubbles entrained at the hydraulic jumps. Parameters: Mean inclination angle: 10°; (a): film thickness: 11.5 mm; (b): film thickness: 13.8 mm.

existence of a bistable branch where surface rollers and shock front exist depending on the flow history. Further decreasing the flow rate, at the lower end of the bistable region, the surface rollers degenerate into a shock front with small, generally instationary, fingering-like structures. The latter seem to pour out of the shock-front and surf on the incoming flow as shown in Figure 2. With decreasing flow rate the fraction that is covered by the fingering also decreases. We quantify the bistable range by the position of the local surface indentation and show how the Froude number for the limits of the bistable branch depend on the inclination angle of the sinusoidal bottom.

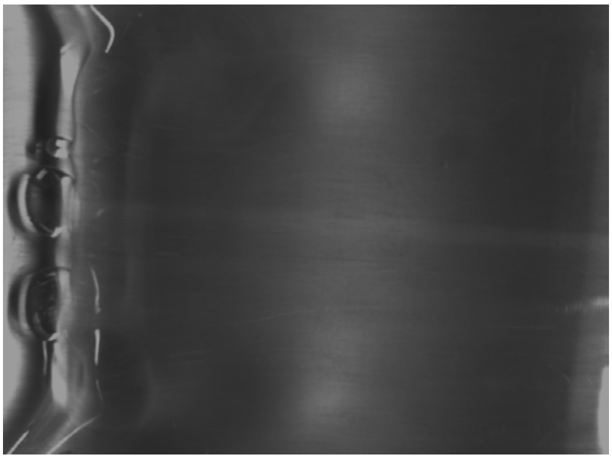

Figure 2: Images of the jump at the lower limit of the bistable. Mean inclination angle: 10°; film thickness: 11.6 mm.

At high flow rates, the surface roller also looses its stability. It is suppressed by standing waves that are in resonance with the bottom contour. Examples of the latter are shown in Figure 3. The transition between these two structures happens in a small parameter region. Here, the system switches periodically between surface rollers and standing waves. We have observed frequencies of about *0.1 Hz* to *0.2 Hz*, depending on the inclination angle. Although kinematic surface waves come in from the flat incline at the Reynolds numbers where the transition takes place, they do not seem to be responsible for the switching. They rather seem to accelerate or retain the switching as a small perturbation, leading to a broadening in the frequency spectrum.

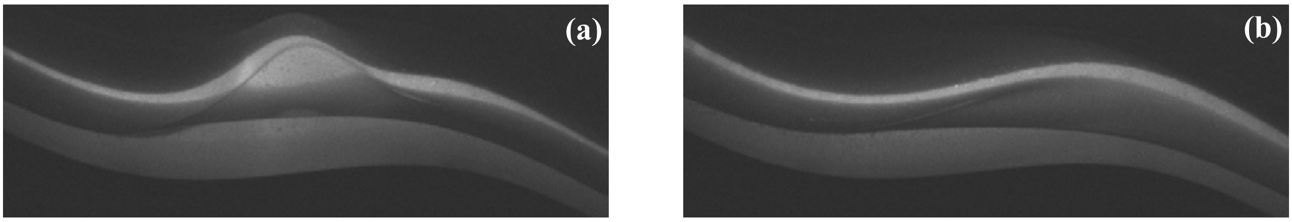

Figure 3: Side-view of resonant standing waves. The free-surface contour corresponds to the upper limit of the bright central part. Mean inclination angle: 12°; film thickness: 13.6 mm (a) and 14.7 mm (b).

CONCLUSIONS

We observed a bistability between shock fronts and surface rollers. At the low end of the bistable region the surface rollers degenerate into an instationary fingering-like structure pouring out of a shock front. It seems to us that surface tension plays an essential role for the existence of the bistability. At higher flow rates the hydraulic jumps loose their stability in favor for standing waves that are in resonance with the bottom contour. At this transition, we observe a switching of low frequency between surface rollers and standing waves.

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