ANALYTICAL MODELS FOR SHOCKS IN TRANSONIC FLOW

Helmut Sobieczky

DLR German Aerospace Center, Bunsenstr. 10, Göttingen, 37073 Germany

Summary Objective is trying to maintain the value of analytic modelling for deeper insight, education and development of new design concepts. Analytical flow models are used to confirm wellknown and find new solutions: Logarithmic structure of surface pressure distribution occurs with occurence of different shock waves interacting with contour geometry.

MAPPING THE FLOW PROBLEMS TO THE HODOGRAPH

A recent IUTAM symposium [1] on transonic flow has shown that analytic treatment of complex flows still draws interest as an efficient tool to shed light into complex flow details where numerical results give no clear answers.
Complexity of flow problems may stem from special boundary conditions as well as from dynamics within the flow field. In transonic flow, the latter source of complexity, for instance, occurs due to shock waves within the flow field. This holds already for steady, inviscid 2D flow. Aerodynamic applications, of course, require the simulation of steady and unsteady viscous and 3D flows, but with recent developments in flow control technology, as seen in [1], prescribing target pressure distributions to control viscous interaction becomes important. We are therefore interested to preserve some of the findings from inviscdid, 2D flows models, guiding us to suitably parameterize design parameters.
Here we use the idealized 2D near sonic flow equations (1) for velocity components U, V desribing perturbed sonic velocity parallel flow (U = 0, V = 0). These equations model potential flow for vorticity $\omega = 0$ and also represent a small perturbation approximation of the Euler equations. For $\omega = 0$ the hodograph transformation to velocity variables ν,θ (Prandtl-Meyer angle, $\nu \sim \pm|U|^{3/2}$, and flow angle θ) results in the Beltrami mapping equations (2). We note that this system is linear and weakly singular at $\nu = 0$, it is the basis for near sonic flow phenomena modelling. Investigation of flow phenomena, which do not include the sonic condition $\nu = 0$, but focus around perturbing a special value of ν, lead to simple Cauchy-Riemann (C-R) or wave equations (3) in the hodograph plane. This is equivalent to suitably linearizing (1) around a given value of $U \neq 0$ and this way also obtaining C-R or wave equations in the (X,Y) physical plane.

$$U \cdot U_X - V_Y = 0 \qquad U_Y - V_X = \omega \tag{1}$$

$$X_\nu = \nu^{1/3} \cdot Y_\vartheta \qquad X_\vartheta = \pm\nu^{1/3} \cdot Y_\nu \tag{2}$$

$$X_\nu = \pm Y_\vartheta \qquad X_\vartheta = Y_\nu \tag{3}$$

A wellknown example: Normal shock on the curved wall
A series of publications has dealt with the problem of a normal shock on a curved wall. A study of the relevant milestones which confirmed the analytical structure of a logarithmic solution for the pressure distribution along the wall, cp ~ a+bXlog(X)+..., should start with the work of v.Koppenfels [2] about incompressible flow past walls with curvature jumps. Emmons [3] numerically solved the compressible flow Euler equations and mentions qualitative relations of his results to flows as investigated by v.Koppenfels. Gadd [4] clearly points out this earlier work to arrive at a logarithmic model, Oswatitsch and Zierep [5] confirm the logarithmic solution by studying the above equation (1), linearized around the value of U behind a normal shock and without vorticity. Final doubts about validity of such local potential solution, based on the occurrence of shock-curvature-induced vorticity, are removed by Fung [6] concluding that the vorticity terms are of a higher order than those governing the logarithmic character of the solution.
Finally, our own work in transonic airfoil design fits to this model, if a shock is being designed using the Fictitious Gas technique: Sobieczky and Niederdrenk [7] use subsonic initial boundary conditions with a curvature jump to solve an elliptic ("fictitious") boundary value problem, before in a second step a supersonic flow pattern terminated by a shock wave and the smoothly curved contour wetted by supersonic flow is constructed by the method of characteristics and the shock relations. Such academic examples led us to develop novel wing design strategies, to be applied subsequently to 3D, viscous and even unsteady aerodynamics. Examples are shown in [1].

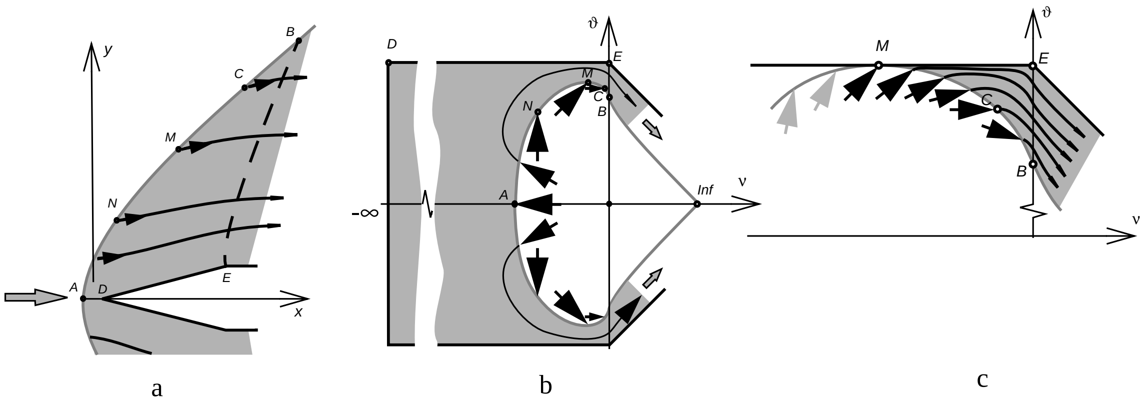
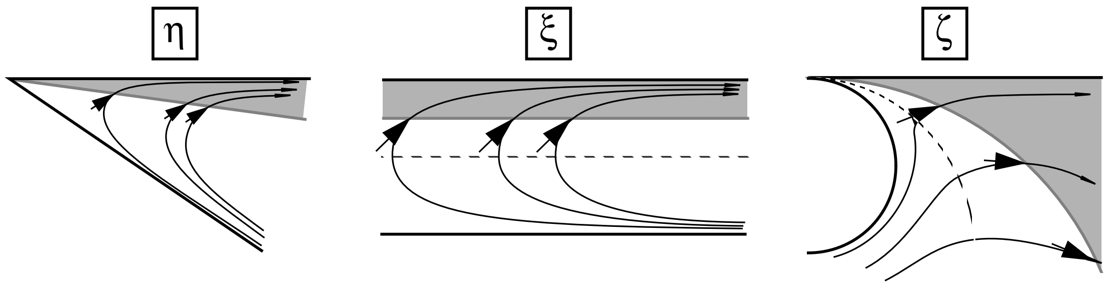


Fig. 1: Shock attachment to a wedge in low supersonic Mach number flow (a), analog to growing shock polar with flow emanating under given directions in hodograph plane (b) until touching fixed contour DE, (c).



$$Z = X - iY = \eta^n \qquad \eta = e^{\xi} \qquad \zeta = (\nu - \nu_M) + i(\vartheta - \vartheta_M) = 1/\xi$$

Fig. 2: Conformal mappings to find solution ζ = Fct(Z), resulting in wedge surface pressure $c_p(\nu) = \mathrm{fct}(X)_{Y=0}$

Another logarithmic flow singularity: shock attaching to a wedge airfoil leading edge

Here another problem is presented involving the shock relations near a singular location on the boundary: Shock attachment to a wedge leading edge (Fig. 1a) requires a transition from a detached normal shock to an oblique shock. This problem is treated easier in the hodograph plane (Fig. 1b) using the Beltrami mapping system (2), the shock polar and investigating the vicinity of the shock polar maximum deflection point M (Fig 1c). This allows for linearization around the subsonic ν_M using C-R system (3). With systems (2) and (3) representing analog flow in the hodograph plane (ν,θ), we have to describe a potential flow model in the sharp angle left of M, with a straight solid boundary above and a curved transpirating boundary with flow emanating under 45^o from it. For solution of this boundary value problem a series of simple conformal mappings is used and illustrated in Fig. 2. From these the pressure coefficient along the wedge contour near the tip results to $c_p \sim a+b/\log(X)+$....., obviously a hitherto not yet described detail of shock attachment to a wedge.

CONCLUSION

Analytical flow models are still seen as precise tools to: (1) understand complicated flow patterns generated by special boundary conditions, (2) subsequently explain and educate about flow phenomena and (3), finally use their suitably parameterized mathematics for aerospace design and optimization.

References

1 Sobieczky, H. (Ed.): Proc. IUTAM Symposium Transsonicum IV. Kluwer Acad. Publ., Dordrecht 2003
2 v.Koppenfels, W.: Luftfahrtforschung Bd. 17, Lfg. 7, (1940), translated in NACA TM 996, 1941
3 Emmons, H. W.: Flow of a Compressible Fluid past a Symmetrical Airfoil. NACA TN 1746, 1948
4 Gadd, G. E.: The Possibility of Normal Shock Waves on a Body with Convex Surfaces in Inviscid Transonic Flow. *ZAMP* **11**, 1960
5 Oswatitsch, K., Zierep, J.: Das Problem des Senkrechten Stosses an einer Gekrümmten Wand. *ZAMM* **40**, 1960
6 Fung, K.-Y.: Vorticity at the Shock Foot in Inviscid Flow. *AIAA Journal* **21**, No. 6, 1983
7 Sobieczky, H., Niederdrenk, P.: Entwurf einer Transsonischen Strömung mit Verdichtungsstoss *ZAMM* **65**, 1985.