XXI ICTAM, 15-21 August 2004, Warsaw, Poland

HYDRODYNAMIC EFFECT OF SLOW PHASE TRANSITIONS IN MICROGRAVITY

Eleonora K. Bevza[*], Olexander O. Kochubey[*], Dmytro V. Yevdokymov[*]
[*]*Dniepropetrovsk National University, Mechanics and Mathematics Faculty, Nauchna str. 13, Dniepropetrovsk, 49050, Ukraine*

Summary The hydrodynamic effect of slow phase transitions is investigated using small parameter method. The mentioned effect is caused by density difference in phase transformation. Analytical and numerical calculation methods are developed for the proposed mathematical model. Estimations of correspondent convective flux influence on phase transition process are obtained.

STATEMENT OF PROBLEM

In general case origin and created phases have sufficiently different densities (case of equal densities is extremely seldom), what leads to fluid flows in liquid phases. Especially large this difference in evaporation (boiling) and condensation processes. Consider, as an example, an evaporation process. Created gaseous phase initially occupies the same volume, what was occupied by the transformed liquid phase, but it means that the created gaseous phase is strongly compressed and, as a result, the pressure in the created volume of new phase is more than in the rest of gas. This pressure difference induces gas flow which tends to equate the pressure in the whole gas phase and therefore the density too. On the other hand, the liquid phase can move too due to the same high pressure action, however, since the liquid density is much more, the gas density under the same conditions, the velocity of the liquid phase is smaller than gaseous one. Similar effects take place for other kind of phase transitions.

There are a lot of specific features in heat and mass transfer during phase transformation in microgravity condition. The main difference between heat and mass transfer in microgravity condition and in terrestrial condition is, of course, absence of natural convection. In the present work an absence of the forced convection is assumed, because forced convection is rather seldom phenomena in microgravity. By these reasons the process is mainly determined by heat conduction or diffusion phenomena, but influence of several other effects can be sufficient too. The main aim of the present work is restricted by slow phase transformations, that is, by the case of quite small gradients of parameters, which determined the phase transition. Such situation is enough widespread onboard of space vehicles and in some other cases of microgravity.

Consider a general mathematical model of phase transition:

$$\frac{\partial T_1}{\partial t} + V_1 \mathrm{grad} T_1 = a_1 \Delta T_1, \tag{1}$$

$$\frac{\partial T_2}{\partial t} + V_2 \mathrm{grad} T_2 = a_2 \Delta T_2, \tag{2}$$

Where V_1, V_2 are fields of velocities in the first and second phases, T_1, T_2 are fields of temperatures in the first and second phases. For the sake of simplicity let prescribe first kind boundary conditions on the outer boundary of phases

$$T_1\big|_{\Gamma_1} = T_{1s}, T_2\big|_{\Gamma_2} = T_{2s}, \tag{3}$$

and the initial conditions

$$T_1(t=0) = T_{10},\ T_2(t=0) = T_{20} \tag{4}$$

The following conditions take place on phase transformation boundary:

$$T_1\big|_{\Gamma_{p.t.}} = T_{p.t.},\ T_2\big|_{\Gamma_{p.t.}} = T_{p.t.}, \tag{5}$$

and the Stefan condition

$$\lambda_1 \frac{\partial T_1}{\partial n}\bigg|_{\Gamma_{p.t.}} - \lambda_2 \frac{\partial T_2}{\partial n}\bigg|_{\Gamma_{p.t.}} = \sigma\rho \frac{\partial n}{\partial t}, \tag{6}$$

where ρ is density of one phase (usually incompressible). Note that the right hand part of relation (6) is not full velocity of interphase boundary propagation, but it is only its component corresponding to phase transformation. The full velocity of interphase boundary is described by following relation

$$V_{p.t.} = \frac{\partial n}{\partial t}\bigg|_{\Gamma_{p.t.}} + V_h, \tag{7}$$

where V_h is hydrodynamic velocity of interphase boundary. As a result, the interphase boundary motion is described by equation

$$\frac{\partial \eta}{\partial t} = V_{p.t.}, \tag{8}$$

here η is generalized coordinate of interphase boundary referred along a normal to it, $V_{p.t.}$ is determined from (7). The equation (8) should be supplemented by the initial condition reflecting initial position of the interphase boundary:

$$\eta(t=0)=\eta_0 . \tag{9}$$

However, above formulated boundary-value problem is non-completed, as it doesn't contain an algorithm of velocity field determination. It is specific hydrodynamic problem. Traditionally as a model of hydrodynamics in heat and mass transfer processes Navier-Stokes or Reynolds equations are used. However, there isn't a tangent velocity component on the interphase boundary in the considering problem, that is, if there is not any other flow in the liquid domain, the flow caused by phase transition can be modeled by potential ideal fluid flow, what is much easier. On the other hand, if there is any other flow, viscous liquid model must be applied. Other specific feature of the considered problem is low speeds of flow, that is, the fluids in both phases can be considered as incompressible $\operatorname{div} V_1 = 0$, $\operatorname{div} V_2 = 0$.

Let the problem is described by Navier-Stokes equations or Euler equations formulated by traditional way. Let prescribe boundary conditions for the problem (restrict the consideration by interphase boundary, because traditional conditions are prescribed on other parts of boundaries). Equality of tangential velocity is assumed there (for Navier-Stokes equations)

$$V_{1\tau}\big|_{\Gamma_{p.t.}} = V_{2\tau}\big|_{\Gamma_{p.t.}} , \tag{10}$$

(or $V_{1\tau}\big|_{\Gamma_{p.t.}} = V_{2\tau}\big|_{\Gamma_{p.t.}} = 0$ for Euler equations). The normal velocity components are coupled by relation

$$|V_1| + |V_2| = \left(\frac{\rho_2}{\rho_1} - 1\right)\left|\frac{\partial n}{\partial t}\right| . \tag{11}$$

Pressure equality condition must be added

$$P_1\big|_{\Gamma_{p.t.}} = P_2\big|_{\Gamma_{p.t.}} . \tag{12}$$

The problem can be can be simplified, if it is neglected by motion of one phase. Consider, for example, one-dimensional case of one moving phase, described by equation

$$\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial x} = a\frac{\partial^2 T}{\partial x^2} , \tag{13}$$

with correspondent boundary and initial conditions. The following relations are important among boundary conditions

$$\lambda \frac{\partial T}{\partial n}\bigg|_{x=y} = \sigma\rho_2 \frac{dy}{dt} , \tag{14}$$

$$V = -\left(\frac{\rho_2}{\rho_1} - 1\right)\frac{\partial y}{\partial t} , \tag{15}$$

where y is coordinate of phase transition, ρ_1 is density of pair, ρ_2 is density of liquid. Thus

$$\frac{\partial T}{\partial t} - \left(\frac{\rho_2}{\rho_1} - 1\right)\frac{dy}{dt}\frac{\partial T}{\partial x} = a\frac{\partial^2 T}{\partial x^2} . \tag{16}$$

Correspondent dimensionless form of the problem is

$$\frac{\partial \theta}{\partial t_{st}} St + W\frac{\partial \theta}{\partial x} = a\frac{\partial^2 \theta}{\partial x^2} , \tag{17}$$

where $St = \dfrac{cp(T_n - T_{p.t.})\rho_1}{\sigma\rho_2}$ is Stefan number, W is dimensionless velocity. It is easy to show that equation (17) is transformed to

$$St\left(\frac{\partial \theta}{\partial \tau_{St}} - \left(\frac{\rho_2}{\rho_1} - 1\right)\frac{d\eta}{d\tau_{St}}\frac{\partial \theta}{\partial x}\right) = \frac{\partial^2 \theta}{\partial x^2} . \tag{18}$$

Application of small parameter method (Stefan number is the small parameter) to equation (18) with dimensionless boundary and initial conditions gives an opportunity to solve the problem analytically. Note that convective term doesn't influence on zero approximation.

The proposed approach is easy generalized to two- and three-dimensional cases, the correspondent boundary-value problems are solved by boundary element method.