GRAVITY FLOW OF A DENSELY-PACKED GRANULAR MATERIAL.

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Summary We present experimental results concerning the rapid flow of a densely-packed grain collection down a bumpy inclined channel. We show that the results do not agree with the predictions of the standard kinetic theory, relying on the binary collision hypothesis. Emphasizing the role played by multicontact collisions in the dense limit, we propose a new approach relying on a long range dissipation scheme. Our model succeeds in accounting qualitatively and quantitatively for the linear profile of velocity found in experiments on dense gravity-driven flows.

INTRODUCTION

So far, there is a reasonable qualitative agreement between the results of experiments conducted on dilute granular media and the predictions of kinetic models inspired from the hard sphere gas theory [1]. However, experiments conducted on dense rapid granular flows lead to drastically different results, results which appear to lie beyond the domain of validity of standard hydrodynamic descriptions [2]. The reason is likely that for dense media, there are long-lasting contacts between grains, and that the basis hypothesis of binary collisions does not hold. Contrary to dilute granular gases, for which the transport of momentum proceeds from balistic flights and interparticle collisions, the most efficient channel to transport momentum and energy through dense granular media is supported by the continuous paths of contacts.

EXPERIMENTAL RESULTS

We have performed experiments in a two-dimensional inclined channel, sloping between 20° and 30°. The bottom is constituted of a saw blade which ensures non-slipping boundary conditions at the bottom for weak enough slopes. The granular material is comprised of monodisperse aluminum spheres of 1.5 mm diameter, with elastic restitution coefficient e =0.6 and friction coefficient μ=0.6. The flows are filmed with a high speed digital camera (250 frames/sec) and then pictures are analysed to access grain position and velocity fields. A typical picture of the flow is shown in Fig. 1. Corresponding solid fraction and velocity profiles are shown in Fig. 2. The salient features are the followings. Firstly the solid fraction appears as nearly constant in the flowing layer (Fig. 2), with value $\nu \cong 0.8$ corresponding to the random close packing (except for the very upper region, owing to the unevenness of the free surface) in two dimensions. Secondly, the shear rate $\dot{\gamma}$ is found to be independent of the depth, i.e. the velocity profile is linear (Fig. 2), and its order of magnitude is given by $\sqrt{g/d}$, where g is the gravity constant and d is the grain diameter. Interestingly, using other materials (as steel beads), the rheological behavior is found to be insensitive to the value of the restitution coefficient e. As for the instantaneous velocity fluctuations in densely-packed materials, it is important to realize that reliable data are very difficult to obtain experimentally. Real experiments can access to grain trajectories only to within the experimental time resolution, and the ostensible velocity fluctuations result more from sliding of grains over adjacent corrugated layers of particles than from the ballistic flight of grains punctuated by distinct collisions. Sampling of grain displacements over a time very short compared with the characteristic shear time ($\nabla v)^{-1}$ indicates an approximatively constant value for the velocity fluctuations through the flow.

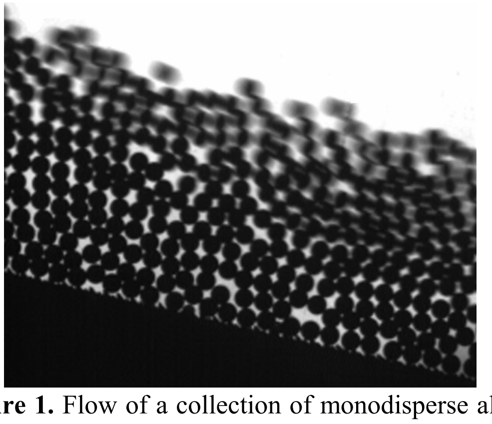

Figure 1. Flow of a collection of monodisperse aluminum spheres (restitution coefficient e=0.6, flow rate 1100 grains/sec, exposure time of the photograph : 1/125 sec). .

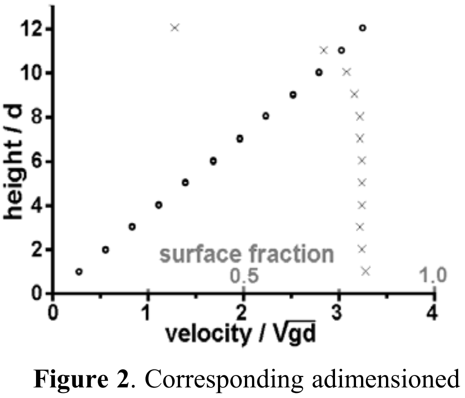

Figure 2. Corresponding adimensioned velocity profile and solid fraction (averaged over 100 samplings).

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The above observations cannot be accounted for by kinetic theory. Indeed, for such constant-density, isothermal flows, the kinetic theory recovers the Bagnoldian quadratic dependence of the shear stress on shear rate [3], and hence leads to the following stress balance $(\partial v_x/\partial z)^2 \propto \rho g z \sin\theta$ (where z is oriented downwards, normal to the flow and x is the direction of the flow) in the steady regime . One therefore obtains $v_x \propto (\rho g h^3 \sin\theta)^{1/2} [1 - (z/h)^{3/2}]$ and the 3/2-power law with respect to the depth disagrees the linear velocity profiles found experimentally in dense media.

PROPOSED INTERPRETATION

As mentioned above, this discrepancy likely proceeds from the inadequacy of the binary collision picture in the case of dense materials. Collisions of solids involve various phenomena such as deformations, waves, heating, etc...[4]. The *coefficient of restitution* is well defined only for direct collisions of free, unconstrained spheres, and this is improper to extend it to the case of multibody collisions [5]. It is worth noting that in the case of multibody collisions, the apparent restitution cefficient can be considerably smaller than the one defined from binary collisions.Recently, Falcon *et al.* [6] showed experimentally (in a 1-d geometry) that a column of spherical grains colliding as a whole with a bottom wall, displayed a bounce height decreasing strongly as increasing the number of grains constituting the column. For such *multibody collisions* (with grains subjected to unilateral constraints, owing to noncohesiveness), the apparent restitution coefficient is seen to be zero as the number of the close-packed grains overpasses a threshold value. The reason is that a mechanical wave propagates through the chains of grains in contact, and experiences partial reflection and dissipation through each grain interfaces. Over a certain size (depending on the dimensionality) both momentum and kinetic energy are fully damped in a finite time τ_{damp}, so that the *apparent restitution coefficient* of the impacting column *appears equal to zero*. This result meets the inelastic collapse phenomenon which arises from the modeling of collision waves experiencing dissipative impacts [7]. We advance that taking into account multicontact collisions and the subsequent apparent decrease to zero of the apparent restitution coefficient observed in thick granular bed, are crucial to explain the behavior observed in densely packed gravity flow. We do not plan here to describe in detail the propagation of deformation waves through the bulk. We just specify that the relative momentum and the kinetic energy gained by each grains between two consecutive collisions are fully dissipated, consistently with the result of Falcon *et al.,* through the bulk in a very short time τ_{damp}, compared to the shearing time $(\partial v_x/\partial z)^{-1}$. The spirit of our reasoning is the following. Let us adopt a Lagrangian description, and consider a pair of virtually colliding particles belonging to two adjacent layers. The time-averaged increment of velocity $\bar{v}$ gained by the upper grain between two successive collisions reads as $\sqrt{gd \sin\theta}$ (where θ is the slope). We derive readily $\partial v_x/\partial z = \bar{v}/d = \sqrt{g \sin\theta / d}$. Note that this steady state solution results from the following momentum equation (per unit mass) $Dv/Dt = g \sin\theta - d (\nabla v)^2$ (D/Dt is the material derivative) where the damping force $F = -d (\nabla v)^2$ results from the averaged momentum given in to the bulk. Then, introducing the Coulomb friction term between layers, we get the following momentum equation: $Dv/Dt = g \sin\theta - kg \cos\theta - d (\nabla v)^2$ (where $k = \tan\theta_c$ is the coefficient of friction of the material), and the shear rate in the steady regime consequently reads as

$$\frac{\partial v_x}{\partial z} = \left[\frac{\sin(\theta-\theta_c)}{\cos\theta_c} \right]^{1/2} \sqrt{\frac{g}{d}}$$

in agreement with the observed linear profile of velocity, and with a prefactor of correct magnitude. Note that the previous dependence of the shear rate was also recognized experimentally by Orpe and Khakhar in the rotating drum geometry [8]. It is worth noting that the paradoxical nonzero shear rate which is experimentally observed at the vicinity of the free surface, opposes the predictions of standard hydrodynamic theories, but is recovered within our approach. The reason is that, for *constrained* collisions, the momentum is not conserved in the center-of-mass frame of the virtually colliding particles [5].

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