MOTION AND STABILITY OF AN ELASTIC HEAVY TOP

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Abstract. Motion and stability of an elastic top with stress-free surface and a fixed point moving in a gravitational field is considered. The investigation is based on a direct approach in the energy momentum method split into two steps.Relative equilibrium states are determined and criteria for stability are proved.The obtained results are applied for the case of a sleeping heavy top when the top is an elastic circular cylinder.

DEFORMATION AND MOTION OF THE TOP

In this work we consider an elastic and asymmetric top with stress-free surface and a fixed point moving in a gravitational field. It is assumed that the elastic deformation of the top remains static during the motion. The literature on the problem when the top is rigid one is well known [1]. When the top is elastic one the problem becomes very complicated since the Hamiltonian system is infinite dimensional with respect to the elastic deformation whereas the motion of the deformed top as a rigid one is only determined by three independent variables. In order to solve this problem we split the solution into two steps. First we look for the static elastic deformation applying a direct approach in the energy momentum method with energy momentum functional which contains six independent variables at each point of the top. The next step is to rewrite the same Hamiltonian in such a way that the energy momentum function to contain only the independent variables that describe the motion of a rigid top. These two steps are connected each other by the angular velocity of rotation. When we consider the deformation of the top we assume that it depends not only on the gravitation but also on the rotation of the top with arbitrarily fixed angular velocity. In the next step we consider the deformed top as a rigid one. The deformation introduces the angular velocity via the components of the inertia tensor which we do not vary assuming now that the deformation is fixed but we vary the angular velocity when it enters in the Hamiltonian in connection with the rotation of the top as a rigid one. The result is that the position coordinate - the nutation and the cyclic velocity - the precession velocity are constants (there are no proper rotation) and therefore the motion is so called relative equilibrium. The existence of relative equilibria is connected with the fact that the top possesses a conserved quantity as consequence of a symmetry property.This conserved quantity has been included in the formulation of the energy momentum functional[2-4].

STABILITY OF THE TOP

We define that the state of the deformed elastic heavy top is stable at a relative equilibrium if it is in a stable state with respect to the internal(material) stability connected with the static deformation of the top and the external (rigid) stability due to the rotation of the top as a rigid one. Instead of the definition for formal stability adopted in Simo [2] the Koiter's definition for nonlinear stability with respect to the deformation [5] has been accepted. It is based on the Lyapunov ideas for stability developed for mechanics of continua. The advantage of this definition is in the properly chosen measures for the initial disturbance and the distance to the relative equilibrium state. The usual Lagrange definition for stability of the motion of the deformed top as a rigid one is adopted. On this basis necessary and sufficient conditions for internal and external nonlinear stability are proved for the determined relative equilibrium states. These stability conditions are more complicated than the corresponding one for a rigid top since they depend on the deformation of the top due to its rotation and the gravitation. Detailed consideration of this coupled problem is presented and discussed in this work.The obtained results are applied for the case of a sleeping heavy top when the top is an elastic circular cylinder.

References

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