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REMOTE RECOIL AND WAVE CAPTURE: WAVE–VORTEX INTERACTIONS IN ATMOSPHERE–OCEAN MODELS

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Summary In a recent paper [1] we described a fundamentally new wave-mean or wave-vortex interaction effect able to force persistent, cumulative change in mean flows in the absence of wave breaking or other kinds of wave dissipation. It is associated with the refraction of nondissipating waves by inhomogeneous mean (vortical) flows. The simplest relevant case is that of a narrow beam of sound waves, or shallow-water gravity waves, weakly refracted by a single vortex in two dimensions. An effective recoil force arises. This accords with expectation from a naive photon analogy or "pseudomomentum rule" *except* that the force acts not where the waves refract, but at the vortex core, even if the core is spatially separated from the refracting beam of waves. Strong refraction brings further phenomena including catastrophic "wave capture", a nontrivial variant of classical critical-layer absorption; see the more detailed presentation on this by Bühler in this Minisymposium. One implication is that there are missing forces not yet accounted for in atmospheric climate and weather-prediction models. Connections with the "pseudomomentum rule" and the "wave momentum myth" are discussed.

INTRODUCTION

Wave-induced eastward or westward mean forces and the consequent "gyroscopic pumping" drive persistent, global-scale, greenhouse-gas-transporting meridional circulations in the Earth's middle atmosphere, between about 15 and 100 km altitude. In the middle latitudes of the summer hemisphere, for instance, persistent eastward forces mediated by internal gravity waves pump air equatorwards, through the Coriolis effect, at mesospheric altitudes around 90 km; for a recent review see [2]. The resulting polar upwelling supplies water vapour from below and acts as a gigantic natural refrigerator. Despite intense solar radiation, maximal at the pole because of the Earth's tilt, temperatures as low as 105 K have been observed in the summer polar cap. These are by far the lowest of naturally-occurring terrestrial temperatures: to that extent, the sunniest place on Earth is also the coldest place on Earth.

The wave-induced forces responsible for this remarkable refrigeration phenomenon — and other equally remarkable phenomena such as the "quasi-biennial oscillation" of the east–west winds in the tropical lower stratosphere — cannot all be directly represented in global numerical models of the atmosphere. The reason is the small spatial scale of the waves, many of which are not resolvable by the global-model numerics. Therefore standard practice today is to represent the waves through so-called "parametrization schemes" based on ray theory and wave–mean interaction theory. However, the version of the wave–mean theory on which all such schemes are based is the classical version that neglects horizontal refraction, in which the only persistent wave-induced forces are those associated with wave breaking and other wave-dissipation processes. Lifting this restriction reveals a new set of interesting wave–mean interaction phenomena.

EFFECTS OF HORIZONTAL REFRACTION

There are two main points. First, a generalized wave–mean theory that includes horizontal refraction implies the existence of persistent wave-induced forces of a fundamentally different kind from those considered in classical wave–mean theory. These "missing forces" are unrelated to wave dissipation, yet have a cumulative impact. They may furthermore involve interesting "remote recoil" effects, one of which is that the effective wave-induced force, for wave parametrization purposes, may act at a location removed from the locations where the waves refract. The simplest relevant model problem is the refraction of a narrow beam of acoustic or gravity waves by a single vortex, in a two-dimensional gas-dynamic or shallow-water system. There, the effective force is felt at the vortex core, regardless of its spatial separation from the waves. The reality and precise mechanism of this remote recoil effect is clearly demonstrated, in detail, by the analysis given in ref. [1]. That such remote recoil effects are, moreover, generic is clear from the results in [1] taken together with classical studies of internal gravity waves, notably that of Bretherton [3].

Second, horizontal refraction, a seemingly slow process on naïve order-of-magnitude grounds, for internal gravity waves in typical background wind fields in the atmosphere, can nevertheless accelerate catastrophically in a significant proportion of cases through a process that may be called "wave capture". This is a nontrivial variant of classical critical-layer absorption, in which the magnitude $|\mathbf{k}|$ of the wavenumber vector $\mathbf{k}$ increases not linearly with time, as in the classical case, but exponentially. The reason for the exponential behaviour is the diminution to zero of phase and group velocities as $|\mathbf{k}| \to \infty$, so that the wavecrests behave asymptotically like passive tracers in a horizontal deformation field. Note that in this situation $|\mathbf{k}| \to \infty$ because the *horizontal components* of $|\mathbf{k}|$ tend to infinity. The wave-capture phenomenon provides some striking variations on the theme of missing forces and remote-recoil effects.

[**STOP PRESS:** after submission of this Abstract, we discovered that the linear theory of wave capture (though not the missing-force and remote-recoil aspects) has already been discussed in a paper by Badulin and Shrira [10] under the heading "wave trapping". There is further discussion in a paper by Staquet and Edwards to this Minisymposium.]

WAVE CAPTURE AND THE WAVE MOMENTUM MYTH

Wave capture provides what at first sight appears to be a further counterexample to "wave momentum myth", the conflation of momentum with pseudomomentum with its long history in which, as the physicist E. I. Blount once wrote, "the list of disputants is very distinguished" [4],[5]. The conflation, moreover, tends to be perpetuated in the sound-wave and water-wave literatures because of the special and peculiar properties of irrotational wave motion [6],[7]. The myth can often be seen enshrined in statements like "the waves gave *their momentum* [sic] to the mean flow."

The pseudomomentum $\mathbf{p}$ of a wave packet is the wave property that often behaves *as if* it were momentum; for this reason, it is also called the quasimomentum. More precisely, wave–mean interaction theory implies that in many cases of interest the following "pseudomomentum rule" holds, here stated in the form used as the basis for atmospheric gravity-wave parametrizations. The rule states that persistent mean forces arise only where the waves break or otherwise dissipate, and that these wave-induced mean forces are the same *as if* [5]

1. the fluid were absent, and
2. a wave packet had horizontal momentum equal to the horizontal component of $\mathbf{p}$.

The circumstances in which, and sense in which, the rule is justified, have been clarified in ref. [7], and are further clarified in the present work. Within the standard ray-theoretic approximations, we have the well known formula

$$\mathbf{p} \approx E\mathbf{k}/\hat{\omega}$$

where $\mathbf{k}$ is the wavenumber vector and $\hat{\omega}$ the intrinsic frequency, or frequency seen in a co-moving reference frame, i.e. a frame moving locally with the background flow; E is the (positive definite) energy of the wave packet, seen in the same co-moving reference frame, i.e., the intrinsic wave-energy in the Bretherton–Garrett sense [9], the wave-energy derivable from the linearized equations, as distinct from the pseudoenergy, the (sometimes negative) energy used in plasma physics. This leads to an apparent paradox when wave capture is involved, because of the fact, already mentioned, that $|\mathbf{k}| \to \infty$ with the horizontal components tending to infinity. (In classical critical-level absorption, by contrast, only the vertical component tends to infinity.)

So a wave packet undergoing wave capture in, for instance, a pure-strain horizontal deformation field, would have unbounded $|\mathbf{p}|$, suggesting arbitrarily large forces, far larger than those involved in generating the wave packet. The resolution of the paradox involves further remote-recoil effects and is most simply discussed using the concept of Kelvin impulse, following [3]. This too is discussed in more detail by Bühler in this Minisymposium, in terms of the implied "wave–vortex duality": a captured wave packet becomes a vortex dipole, and the wave–mean interaction problem becomes a nontrivial extension of the standard vortex-interaction problem. Full details are presented in ref. [11].

A point to emphasize, therefore, is that the effective forces related to the pseudomomentum $\mathbf{p}$ act only on the rotational part of the flow derived from a Helmholtz decomposition of the horizontal part of the velocity field. The pseudomomentum rule never has relevance to the irrotational part. This is most clearly seen from the relation between $\mathbf{p}$ and Kelvin's circulation theorem, adumbrated in Rayleigh's *Theory of Sound* and evident from the "generalized Lagrangian-mean" form of wave–mean theory via its exact definition of $\mathbf{p}$ [6]–[8]. In the case of large-scale atmosphere–ocean dynamics the effective forces associated with $\mathbf{p}$ are therefore related to the distribution of Rossby–Ertel potential vorticity [2]. In particular, it is is the apparent contribution of $\mathbf{p}$ to the circulation and potential vorticity that behaves in the simplest way during wave capture, resolving the apparently paradoxical behaviour associated with $|\mathbf{p}| \to \infty$ and bringing in the Kelvin impulse concept. Work toward turning this insight into a practical wave parametrization scheme is being undertaken.

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