FREE SURFACE DEFORMATION IN SUSPENSIONS NEAR A ROTATING ROD

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Summary Concentrated suspensions of solid spheres suspended in a Newtonian fluid are studied in a rotating concentric cylinder apparatus. Concentration profiles and the shape of the free surface are imaged using magnetic resonance. The experimental data are used to validate a three-dimensional finite element code, which incorporates both free surface models and a continuum constitutive equation for the suspension rheology.

INTRODUCTION

Continuum modelling of low Reynolds number suspension flows has progressed considerably in the last decade based on early work by Leighton and Acrivos [1,2] and Nott & Brady [3] Newer generations of these models capture the demixing of suspensions of spheres in a variety of one - and two-dimensional flows including torsional flows in parallel and cone-and-plate geometries [4,5]. Although there are arguments in favour of either of two main modelling approaches [6,5], the diffusive flux model [7] has the advantage in terms of computational speed. We have incorporated a model based on the diffusive flux arguments into a massively parallel, three-dimensional, general purpose, multi-physics, finite element computer code developed at Sandia, GOMA [8]. In addition to solving the coupled heat and momentum transport equations, GOMA supports fully coupled free- and moving-boundary parameterization, making it ideal for manufacturing process modeling. In order to validate the treatment of free-surface problems involving suspensions, we have studied the free surface near a rotating rod, in a classic “rod climbing” experiment. Experimental data was taken with magnetic resonance imaging, which allows the visualization of suspended particle concentration profiles as well as detailed views of the free surface profile. Concentrated suspensions of up to 50% by volume of uniform spheres were subjected to a variety of rod rotation rates. Results showed good agreement between model and experiment. The results show that the interface dips near the rotating inner rod, in agreement with earlier studies [9], and that our model captures this normal-stress induced behavior.

EXPERIMENTS

Nuclear magnetic resonance (NMR) measurements were performed at 1.9 T in an Oxford horizontal bore superconducting magnet with a clear bore diameter of 310 mm, using a custom-made linearly driven RF birdcage probe tuned to the ^{1}H frequency of 80.35 MHz, and a TecMag Libra spectrometer (TecMag Inc, Houston, TX, USA). The suspensions consisted of polymethyl methacrylate (PMMA) spheres nominally 100 microns in diameter suspended in a Newtonian viscous oil. The suspending oil was made to match the density of the particles at 21 °C. The fluid consisted of a solution of potassium iodide (KI) salt (Fischer Chemical, Inc) in water mixed with UCON™ 90,000, a poly-alkylene glycol (Union Carbide Corporation). The fluid produces an NMR signal, whereas the PMMA particles do not; therefore, the particle concentration can be measured quantitatively. Figure 1B shows a typical series of images obtained for a suspension of 50% by volume of solids with the inner rod rotating at 20 rpm. The upper left image depicts the initial condition before the motor attached to the rod is started. Each image is an average of the results over the time it takes to make the image (about 4.5 minutes), and images were taken back to back. The method allows some temporal resolution of the formation of the downward curvature of the interface.

MODEL

A diffusive-flux transport model for particle volume fraction, modified to include anisotropy using a flow-aligned tensor, is used [10]. This tensor is based on a local decomposition of the fluid velocity field into principal flow directions (flow, velocity gradient, and vorticity) and causes anisotropic particle migration. The constitutive equation is implemented in a Navier-Stokes solver based on the Galerkin/finite element method [8], and the problem is solved using a two-dimensional (axisymmetric) geometry with a (swirling) velocity component in the third dimension. A unique feature of the code is the treatment of free interfaces (position unknown) using an Arbitrary-Lagrangian-Eulerian formulation. Figure 1C shows results for a suspension of 50% by volume of 100 micron spheres, at early times during the development of the interface shape.

CONCLUSIONS

A finite element solution to the coupled transport equations using a diffusive-flux-based suspension constitutive equation modified to allow anisotropic diffusion does well in predicting the free surface deformation near a rotating rod due to normal stresses.

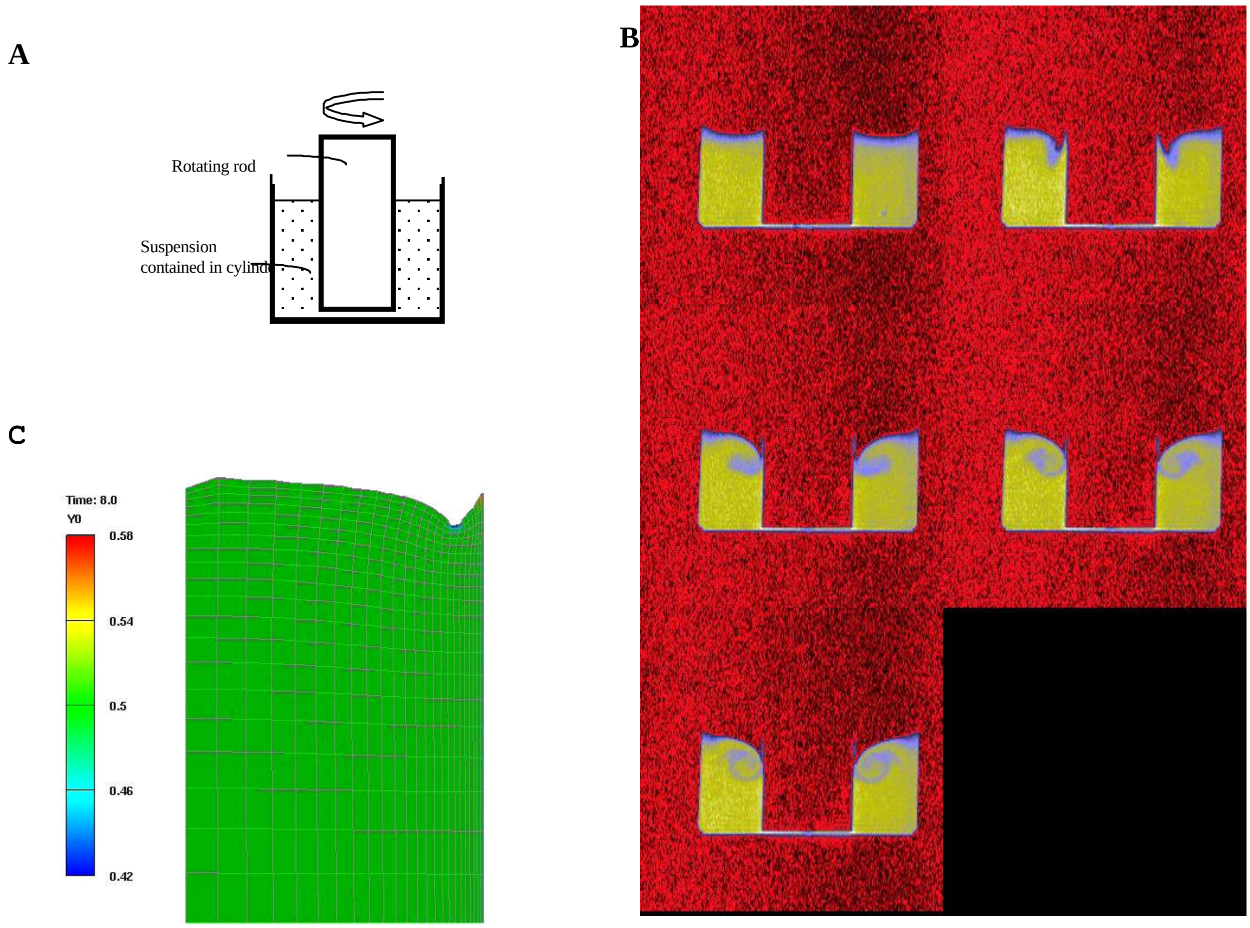

Figure 1. (A) Sketch of rotating rod apparatus. (B) NMR slices of the time evolution (left to right, top to bottom) of suspension liquid concentration. The rotating rod does not produce a signal, so only the suspension is seen. (C) Calculation of half of the domain corresponding to the early time data pictured in (B).

References

[1] Leighton, D., Acrivos, A.: Measurement of Shear-Induced Migration of Particles in Concentrated Suspensions. *J. Fluid Mech.* **177:**109-131, 1987.

[2] Leighton, D., Acrivos, A.: The Shear-Induced Migration of Particles in Concentrated Suspensions. *J. Fluid Mech.* **181:**415-439, 1987.

[3] Nott, P. R., Brady, J. F.: Pressure-Driven Flow of Suspensions: Simulation and Theory. *J. Fluid Mech.* **275**:157-199 (1994).

[4] Morris, J. F., Boulay, F.: Curvilinear Flows of Noncolloidal Suspensions: The Role of Normal Stresses. *J. Rheology* **43**:1213-1237, 1999.

[5] Fang, Z., Mammoli, A., Brady, J. F., Ingber, M. S., Mondy, L. A., Graham, A.L.: Flow-Aligned Tensor Models for Suspension Flows. *Int. J. Multiphase Flow* **28**:137- 166, 2002.

[6] Zarraga, I. E., Leighton, D. T.: Normal stress and diffusion in a dilute suspension of hard spheres undergoing simple shear. *Phys. Fluids* **13**:565-577, 2001.

[7] R. J. Phillips, Armstrong, R. C., Brown, R. A., Graham, A. L., Abbott, J. R.: A Constitutive Equation for Concentrated Suspensions that Accounts for Shear-Induced Particle Migration. *Phys. Fluids A*, **4**: 30-40, 1992.

[8] Schunk, P. R., Sackinger, P. A., Rao, R. R., Chen, K. S., Cairncross, R. A., Baer, T. A., Labreche, D. A.: “GOMA 2.0,” Sandia National Laboratories, SAND97-2404, 1997.

[9] Zarraga, I. E., Hill, D. A., Leighton, D. T.: The Characterization of the Total Stress of Concentrated Suspensions of Noncolloidal Spheres in Newtonian Fluids. *J. Rheol*. **44**:185-220, 2000.

[10] Hopkins, M. M., Mondy, L. A., Rao, R., Altobelli, S. A., Fang, Z., Mammoli, A. A., Ingber, M. S.: Three-Dimensional Modeling of Suspension Flows with a Flow-Aligned Tensor Model. Proceedings of the Pacific Rim Rheology Conference, Vancouver, BC, CANADA July 2001.