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MECHANISMS OF DEEP PENETRATION OF SOFT SOLIDS

Norman A. Fleck* and Oliver Shergold

*Cambridge University Engineering Dept. Trumpington St., Cambridge, CB2 1PZ, UK

Summary Micromechanical models are developed for the deep penetration of soft solids by a flat-bottomed and a sharp-tipped cylindrical punch such as a hypodermic needle. The soft solid represents mammalian skin and silicone rubbers, and is modelled by a one term Ogden strain energy function. The flat-bottomed punch penetration model assumes that penetration is by the formation of a mode II ring crack that propagates ahead of the penetrator tip. The sharp-tipped punch penetration model assumes that penetration is by the formation of a planar mode I crack which is wedged open by the advancing punch. The steady-state penetration load is obtained by equating the work done in advancing the punch to the sum of the fracture work and the strain energy stored in the solid. The penetration pressure for a flat-bottomed punch is three times that for a sharp-tipped punch, in agreement with experimental observations.

INTRODUCTION

The deep penetration of soft solids by a punch is of widespread technological importance, with applications ranging from the piercing of mammalian skin by a hypodermic needle (or by a liquid jet) in administering an injection, to the failure of rubber seals or tires by the penetration of a foreign body. Despite the ubiquitous nature of soft solid penetration, the existing literature provides little insight into the underlying mechanisms of penetration.

MICROMECHANICAL MODELS

Micromechanical models are developed for the deep penetration of a soft solid by a flat-bottomed and by a sharp-tipped cylindrical punch, such as a hypodermic needle. The soft solid is taken to represent mammalian skin and silicone rubbers, and is represented by an incompressible, hyper-elastic, isotropic solid described by a one term Ogden strain energy function for an incompressible, isotropic, hyper-elastic solid,

$$\phi = \frac{2\mu}{\alpha^2}\left(\lambda_1^\alpha + \lambda_2^\alpha + \lambda_3^\alpha - 3\right) \tag{1}$$

where ϕ is the strain energy density per undeformed unit volume, α is a strain hardening exponent and μ is the shear modulus. Table 1 gives measured values of the Ogden constants and toughness J_{IC} for three grades of silicone rubber and human skin used in the deep penetration experiments.

Solid	Grade	μ (MPa)	α	J_{IC} (kJ m^{-2})
Silicone rubber	Sil8000	2.7	2.5	9.1
Silicone rubber	B452	0.4	3.0	7.9
Human skin		0.11	9.0	2.5

Table 1: Curve fits of the Ogden constitutive law (1), and toughness values of some soft solids

The flat-bottomed punch penetration model assumes that penetration is by the formation of a mode II ring crack that propagates ahead of the penetrator tip, see Fig. 1a. The sharp-tipped punch penetration model assumes that penetration is by the formation of a planar mode I crack which is wedged open by the advancing punch, see Fig.1b. Each mechanism has been observed experimentally for both skin and silicone rubbers.

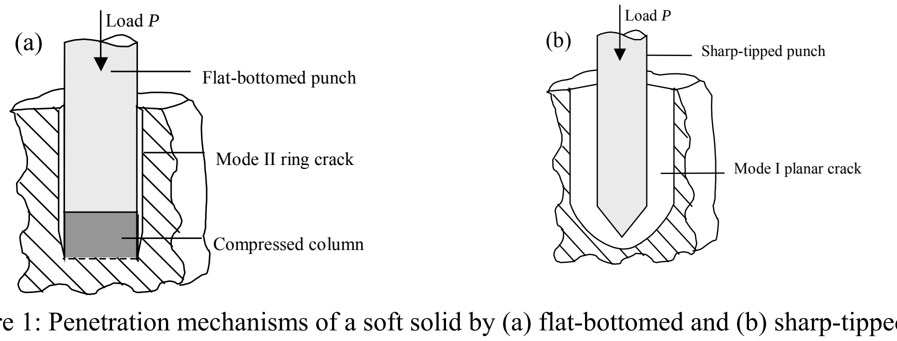

Figure 1: Penetration mechanisms of a soft solid by (a) flat-bottomed and (b) sharp-tipped punch.

Calculation of steady state deep penetration pressure

The steady-state penetration load is obtained by equating the work done in advancing the punch to the sum of the fracture work and the strain energy stored in the solid. For the case of a sharp penetrator, this calculation is performed by considering the opening of a plane strain crack by wedge loading, using a finite element approach. Consider a frictionless, rigid cylindrical punch of radius, R, with a conical tip pushed into a semi-infinite block, as shown in Figure 2a. The solid tears and opens at the tip of the punch. The detailed solution for the punch tip requires a full 3D calculation; however we can consider punch advance by $\delta\ell$ as equivalent to creating a plane strain crack of length $2a$

in a slice of thickness $\delta\ell$ and then opening the crack to accommodate the punch. This energy approach is accurate when the strain energy density in each material element is independent of strain path.
Consider the steady state advance of the punch by an axial increment $\delta\ell$ due to a load P_S. The work done by the punch in effecting this advance is $P_S \cdot \delta\ell$. This work increment balances the energy δW_C required to form a crack of length $2a$ in a solid slice of thickness $\delta\ell$, see
Figure 2b, and the strain energy stored in the solid δS_E on opening the crack to accommodate a circular cylindrical inclusion of radius R, see
Figure 2c. Hence,

$$P_S \cdot \delta\ell = \delta W_C + \delta S_E \tag{2}$$

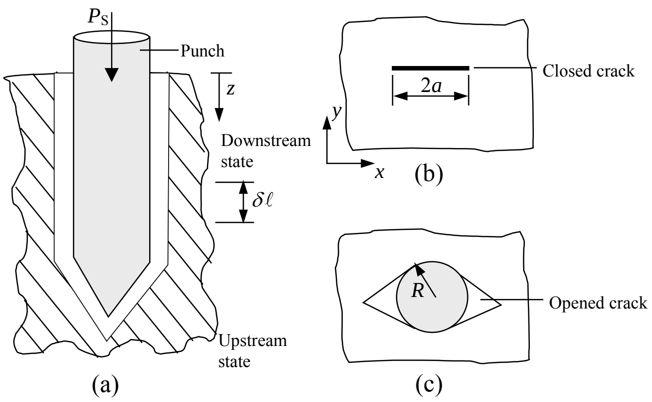

Figure 2: (a) penetration of a soft solid by a sharp-tipped punch, (b) formation of a plane strain crack of length $2a$ in an infinite slice of thickness $\delta\ell$, (c) opening of the crack by an expanding circular cylinder to a final radius R.

The work required to create the crack, δW_C, is determined by the mode I toughness of the material, J_{IC} and is given by

$$\delta W_C = 2 J_{IC} \cdot a \cdot \delta\ell \tag{3}$$

In order to calculate the work S_E required to wedge open the crack, we consider the auxiliary finite element problem of expanding a circular wedge of radius R' from $R'=0$ to a final value of $R'=R$.

Second, consider the case of a flat-bottomed punch. Advance of the punch involves the advance of a mode II ring crack, with compression of a column beneath the punch, as stetched in Fig. 1a. An expression for the stored strain energy in the solid per unit advance of the punch S_E is obtained in closed form, and the punch force is again calculated using relation (2). For both geometries of punch, the crack dimensions are determined by minimising the load on the punch with respect to crack length.

Results

The results for both geometries are shown in Fig. 3. For both geometries of punch tip, the predicted penetration pressure increases with diminishing punch radius, and with increasing toughness and strain hardening capacity of solid. The penetration pressure for a flat-bottomed punch is two to three times greater than that for a sharp-tipped punch (assuming that the mode I and mode II toughnesses are equal), in agreement with experimental observations.

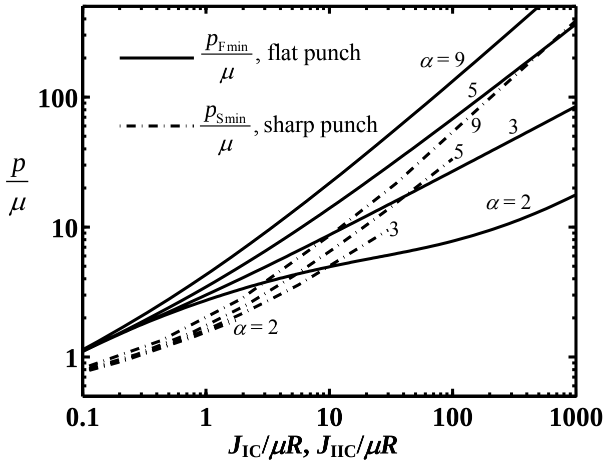

Figure 3: Comparison of p/μ versus $J/\mu R$ for penetration of a solid by a flat-bottomed punch and a sharp-tipped punch.