A UNIVERSAL PROPERTY OF GEOMETRICAL HARDENING

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Summary The main assumptions made in order to quantify geometrical hardening are that the impacts are collinear, and that torques can be ignored; the impact speeds are sufficiently slow so as to be quasi-static; the seismic wave energy losses are insignificant; and the materials are elasto-plastic. Repeated, identical impacts then increase Newton's coefficient of restitution in a universal manner, depending only on the initial coefficient of restitution, and the number of impacts.

INTRODUCTION

Hardening of elasto-plastic materials, due to repeated impacts, is a process made famous from the manufacture of samurai swords. In this process, the metal alters its grain boundary structure, which increases the yield strength of the metal, making it harder, and allowing a finer blade to be manufactured [1].

An analogous process occurs during repeated, identical impacts of elasto-plastic materials, called here "geometrical hardening". This occurs because the repeated, identical impacts increase the area of contact between the two bodies, which increases Newton's coefficient of restitution. Newton's coefficient of restitution is unity for impacts between two plane elasto-plastic bodies, and repeated impacts cause the impacting bodies to tend towards two plane bodies in the region of impact. This then resembles hardening of the metal. However, this is a wholly geometrical effect, and is not associated in any way with a change in the grain structure of the elasto-plastic material.

The aim of the rest of this paper is to detail the universal aspect of how Newton's coefficient increases during the process of geometrical hardening. The main assumptions made in order to derive the results below are that the impacts are collinear, and that torques can be ignored; the impact speeds are sufficiently slow so as to be quasi-static; the seismic wave energy losses are insignificant; and the materials are elasto-plastic.

THE UNIVERSAL CURVE FOR COLLINEAR IMPACTS

The sketch in Figure 1 shows the force-displacement curve for repeated, identical collinear impacts between two elasto-plastic bodies. The first impact occurs with initial and final relative speeds (always assumed collinear) of v ($=v_1$) and w_1. Subsequent impacts occur with the same initial relative speed of v, and separate with a relative speed of w_n. The speeds v_n are the initial relative speeds of approach needed to generate the same displacements as indicated by the points shown as v_n.

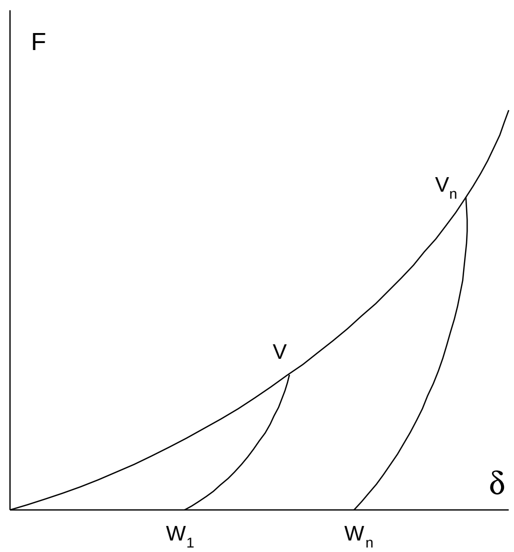

Figure 1. Sketch of the typical force-displacement curve for repeated, identical, quasi-static impacts between two elasto-plastic bodies. The steep path along the w entries is assumed to be wholly elastic, while the upper curve along the v entries is assumed to be essentially fully plastic.

Newton's coefficients of restitution are then C_n, where $C_n = -w_n/v$, and applying an energy balance to the curves in Figure 1 yields

$$C_{n+1}^{\frac{8}{3}} = C_n^{\frac{8}{3}} + C_1^{\frac{8}{3}}[1 - C_n^2] \qquad (1)$$

where we have assumed that the impacts are sufficiently energetic to be in the asymptotic "inverse one quarter" regime,

$$\frac{w_n}{v_n} = -\left(\frac{a}{v_n}\right)^{\frac{1}{4}} \qquad (2)$$

for some constant α [2].

From the recursion relationship in (1), it is clear that as n becomes large, C_n tends towards unity, and so after a great many impacts, the system behaves largely elastically. This is achieved by successively increasing deformations.

Equation (1) is completely independent of particle properties, apart from the appearance of C_1, Newton's coefficient of restitution for the first impact, and so is essentially a universal property of repeated, identical elasto-plastic impacts. The discrete solutions satisfying (1) are given in Figure 2. Given an initial coefficient of restitution on the ordinate, the subsequent increase in the coefficient of restitution is then fixed. Each discrete symbol in Figure 2 corresponds to the solution to the recurrence relationship in (1), with initial conditions running between 0.1 and 0.9 in steps of 0.1. The limiting case in Figure 1 is when w_n is essentially v, since then the impacts simply move the displacements elastically up and down the w lines.

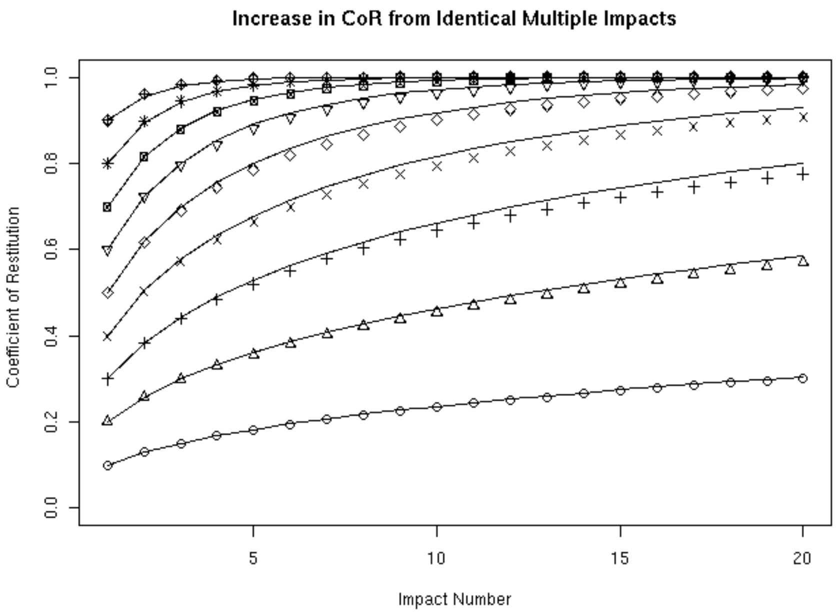

Figure 2. Discrete symbols denote the corresponding solutions to (1), for initial values C_1 between 0.1 and 0.9 in steps of 0.1, as a function of the number of impacts. The continuous lines in Figure 2 are the curves described by (3).

An approximate expression for C_n can be obtained from (1) by setting the term C_n^2 to $C_n^{8/3}$, and then replacing the resulting linear recurrence equation by the corresponding linear differential equation. The corresponding solution is

$$1 - C_n^{\frac{8}{3}} \cong (1 - C_1^{\frac{8}{3}})\exp[-(n-1)C_1^{\frac{8}{3}}] \qquad (3)$$

These continuous curves are plotted in Figure 2, for C_1 running from 0.1 to 0.9 in steps of 0.1. It is clear then that for very low values of C_1, many hundreds of repeated impacts are needed for C_n to approach unity, but for more typical values of C_1, say 0.7, then C_n approaches unity after about 10 or so impacts.

Finally, we note that it is problematical to attempt to quantify the increase in apparent hardening. The approach of Newton's coefficient of restitution to unity suggests the apparent hardening is increasing, whereas the increased displacements which occur suggest that the apparent hardening is reducing. This contradictory situation has developed because while the system is becoming, in practice, more elastic with time, the section of the upper curve in Figure 1 denotes behaviour which becomes more plastic with time.

CONCLUSIONS

The central result is contained in Figure 2, showing that the increase in Newton's coefficient of restitution follows universal curves, depending only on the starting coefficient of restitution, and the subsequent number of impacts. Experimental evidence supporting these universal curves is to appear [3]. In practice, seismic wave losses of a few percent, or so, mean that the universal curves above do not tend exactly to unity as shown, but to a few percent below unity, for quasi-static impacts. Before this happens, experimental difficulties can arise from failure to achieve perfect alignment of the impacting particles, which will cause the experimental measurements of the coefficients of restitution to lie somewhat below the universal curve in Figure 2. Before this happens, there are indications from experiment that the theory in this paper describes the early increases in the coefficient of restitution satisfactorily.

References

[1] Goldsmith, W.: Impact, Edward Arnold Ltd., London, 1960.
[2] Johnston, K.L.: Contact Mechanics, Cambridge Press, 1985.
[3] Weir, G. and Tallon, S.: The coefficient of restitution for normal incident, low velocity particle impacts, submitted to Chem. Eng. Sci, 2004.