SUB- AND SUPERSONIC SHAPES WITHOUT SEPARATION AND CAVITATION

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Summary The slender body theory was applied to calculate axisymmetric bodies with negative pressure gradients at the surface to ensure an unseparated flow pattern and to improve cavitation inception characteristics. A method of calculating of axisymmetric and plane shapes in ideal compressible fluid, based on the potentials of sources and doublets located in the axis of symmetry is proposed. Experimental investigations of axisymmetric shapes with specific pressure distribution are reviewed. Some of such forms ensure unseparated flow pattern at relative small Reynolds numbers. Since the origin of cavitation on a smooth shape is connected with separation, presented bodies could ensure the flow of liquid without cavitation.

AXISYMMETRIC SHAPES WITH NEGATIVE PRESSURE GRADIENTS IN INCOMPRESSIBLE FLUID

There are two ways of obtaining the unseparated forms. The simplest one is to use very slender bodies or very thin profiles, which probably ensure no separation independent of the pressure distribution over the surface. The slender 2D and axisymmetric shapes are investigated in [1] with the use of the Kochin-Loitsiansky method of local similarity (see [2]). A parabolic symmetric profile with the coordinate of the upper surface $Y(x) = 4\varepsilon(1-x)x$ and a body of revolution with the same parabolic generatrix were chosen in [1] to estimate the value of the thickness parameter ε sufficient for the unseparated flow pattern. The calculations show that the separation occurs for very small values of the thickness parameter ε. For example, even for ε=0.01 the coordinates of the separation point are 0.906 (in the 2D case) and 0.912 (in axisymmetric flow). Thus, the simplest way of avoiding the separation is of limited practical interest.

The second way of preventing separation is to use some special forms with appropriate pressure distributions. For example, negative pressure gradients at the body surface are necessary to avoid the separation (see [2,3]). The majority of the researchers consider the minimum of the static pressure coincident with the maximum of the body radius (or with the maximum of the thickness in the 2D case). Moreover, they assume that the pressure gradient is positive after the maximum thickness point. This paradigm was realized in so-called laminarized forms ([3]) obtained by shifting the maximum thickness point as far downstream as possible. It would be interesting to investigate if it is possible to have negative pressure gradients after the maximum thickness point too?

The first approximation of the slender body theory [4] yields the following equation relating the axisymmetric body radius $R(x)$ and the pressure coefficient $Cp(x)$ at its surface (see [1,5]):

$$\frac{d^2R^2}{dx^2} = -\frac{Cp(x)}{ln\,\varepsilon}. \tag{1}$$

To make the trailing edge sharp, discontinuities of dR^2/dx or d^2R^2/dx^2 at some point x_*, $0<x_*<1$ are required (see [6]). According to the first approximation equation (1), the discontinuity of the second derivative d^2R^2/dx^2 provides the discontinuity of the pressure. An example of such a solution obtained in [7] has the following form:

$$Cp(x) = \begin{cases} -ax-2c, & 0 \le x < x_* \\ -a_1(x-1), & x_* \le x \le 1 \end{cases}; \qquad R^2(x) = \begin{cases} Ex^2(ax+6c), & 0 \le x \le x_* \\ Ea_1(x-1)^3, & x_* \le x \le 1 \end{cases}, \quad E = \frac{1}{6\ln\varepsilon} \tag{2}$$

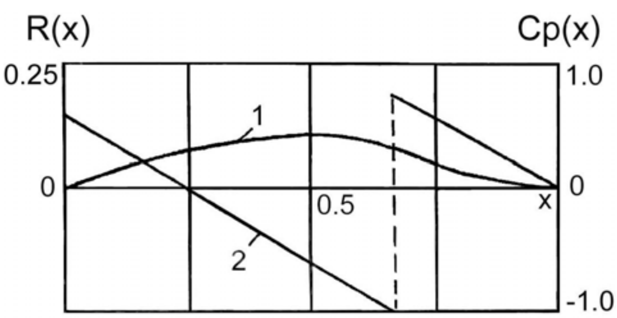

Fig.1 Shape with a negative pressure gradient. First approximation.

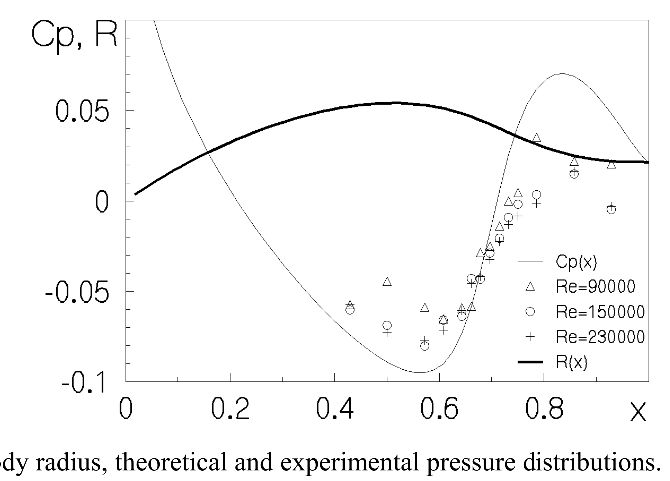

Fig.2. Exact solution. Body radius, theoretical and experimental pressure distributions.

An example of the corresponding axisymmetric shape (curve 1) and the pressure distribution (curve 2) are shown in Fig. 1. To improve the accuracy (eq. (1), (2) are not valid near the point $x = x_*$), some exact solutions of the Euler equations have been calculated, which can be applied to non-slender bodies as well (see [1, 6, 8]). For this purpose, sources and sinks were placed in the axis of symmetry. The first approximation for their intensity $q(x)$ can be obtained from equation (2) and the relationship $q(x) = \pi dR^2 / dx$ (see [2]):

$$q(x) = \begin{cases} 3\pi Ex(ax+4c), & 0 \le x < x_* \\ 3\pi Ea_1(x-1)^2, & x_* \le x \le 1 \end{cases} \tag{3}$$

The appropriate analytical formula for the stream function $\Psi(x,r)$ is presented in [9]. An example of calculations and the experimental pressure distributions for different Reynolds numbers are shown in Fig. 2. The shapes presented in Figs. 1,2 show that the pressure growth downstream of the maximum thickness point is not obligatory. No separation occurred at the body shown in Fig.2 for 90000<Re<300000. Similar 2D profiles with a negative pressure gradient near to the trailing edge are calculated in [1, 9].

AXISYMMETRIC AND 2D SHAPES WITHOUT SEPARATION IN COMPRESSIBLE FLOW

The results presented above can be easily generalized for the compressible flow. Using the stream function of the sources located in the axis of symmetry the following eqs. can be written in sub- and supersonic cases respectively:

$$\Psi(x,r) = 0.5r^2 - \frac{1}{4\pi}\int_0^1 \frac{(x-\xi)q(\xi)d\xi}{\sqrt{(x-\xi)^2+(1-M^2)r^2}}\,; \quad \Psi(x,r) = 0.5r^2 - \frac{1}{2\pi}\int_0^{x-\omega r} \frac{(x-\xi)q(\xi)d\xi}{\sqrt{(x-\xi)^2+(1-M^2)r^2}}\,, \quad \omega = \sqrt{M^2-1} \tag{4}$$

Here M is the Mach number at infinity. It was mentioned in [9] that the first approximation eq. (1) is valid both for a subsonic and for a supersonic flow, therefore substituting relationship (3) into eq. (4) analytic formulae for the stream function can be obtained. The body radius and the pressure distributions at different values of M are calculated.
It was shown in [10] that intensity of sources (3) can be used in 2D case to calculate the form of symmetric profile with negative pressure gradient both upstream and downstream the maximum thickness point, which can ensure unseparated flow pattern as well (unfortunately no wind tunnel tests were performed with such shapes). With the use of the stream function of the 2D sources located in the axis of symmetry similar eqs. can be written in sub- and supersonic cases. The coordinate of the upper surface of the profile and the pressure distributions at different values of M are calculated.

CONCLUSIONS

As the experiments show, the shapes with negative pressure gradients (similar to one shown in Fig.2 and to U-1, [1]) provide the unseparated flow pattern. The presented axisymmetrical and 2D forms are of considerable practical interest because they diminish the total drag (due to the small value of pressure drag and the delay of the laminar-turbulent transition). Using such shapes in water should improve the cavitation characteristics, too, since separation is the main reason of cavitation inception (see theoretical considerations in [1, 11] and experimental arguments in [11]). The water tunnel tests are necessary to investigate the cavity inception characteristics of the presented shapes. Unfortunately, the required equipment is not available in Ukraine. Further experimental investigations of the presented bodies and the profiles at higher Reynolds and Mach numbers are also necessary.

References

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