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ANALYSIS OF GRAZING BIFURCATIONS IN IMPACT MICROACTUATORS

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Summary Impact microactuators rely on repeated collisions to generate gross displacements of a microelectromechanical machine element without the need for large applied forces. Their design and control rely on an understanding of the critical transition between non-impacting and impacting long-term system dynamics and the associated changes in system behavior, known as grazing bifurcations. In this paper, we present three characteristically distinct transition scenarios associated with grazing conditions for a periodic response of an impact microactuator: a discontinuous jump to an impacting periodic response (associated with parameter hysteresis), a continuous transition to an impacting chaotic attractor, and a discontinuous jump to an impacting chaotic attractor. A theoretical normal-form analysis is presented that predicts the character of each transition from a set of conditions that are computable in terms of system properties at grazing. This analysis is validated against results from numerical simulations of a model impact microactuator.

MICROACTUATORS

Precise displacement control and manipulation is required in microscopes, optical devices, nanoscale data storage and during micro surgery. Microdevices are ideal for micropositioning due to their small sizes. Microactuators used to produce small displacements would need large actuation forces and a long driving distance. Actuators based on impulsive forces provide a solution to this problem, as repeated impacts can be used to generate relatively large motion from the accumulation of many small displacements [1]. Recently, impact microactuators have attracted a lot of attention due to ease of fabrication, capability of batch processing, robustness to environment, high accuracy and high power [2, 3, 4, 5]. A schematic of the microactuator studied by Mita *et al* [5] is shown in Figure 1 below. Here, excitation through an applied voltage between the movable block m_2 and electrodes rigidly attached to the frame m_1 results in repeated impacts between the block and the frame (through the stoppers) and resultant brief episodes of sliding motion of the frame. Although each such sliding event contributes only on the order of hundreths of microns of displacement, periodic excitation may be used to generate controlled gross sliding.

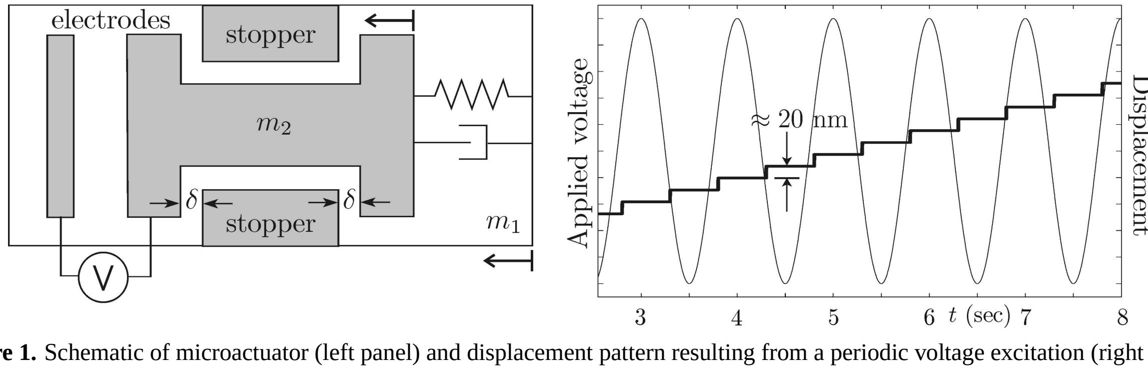

Figure 1. Schematic of microactuator (left panel) and displacement pattern resulting from a periodic voltage excitation (right panel).

Analysis and design of proper function of an impact microactuator relies on an understanding of the possibly dramatic changes in system response that originate at the onset of impacting motions as a system parameter μ (for example, the amplitude or frequency of the driving voltage) is varied. Specifically, we are concerned with bifurcations in the long-term system response that occur as μ increases past some critical value μ_{grazing}, at which value there exists a periodic non-impacting oscillation of m_2 that achieves zero-relative-velocity contact with the stoppers. In a state-space description of the dynamics of the microactuator, such zero-relative-velocity contact corresponds to a *grazing contact* between a state-space trajectory and a discontinuity surface, here representing the sudden change in velocity of m_1 and m_2 that results from an impact. In contrast to periodic trajectories in smooth systems, the local description in the vicinity of a grazing trajectory is well-known to be non-differentiable with dramatic implications to the stability of the grazing trajectory and its persistence under further parameter variations. Indeed, as there is no advance warning of this instability, any local description must account for the nonsmooth character of the flow near the grazing trajectory.

TRANSITION SCENARIOS

Three different transition scenarios are observed in simulations of a periodically forced two-degree-of-freedom model of the impact actuator as shown in Figure 2 below. We note, in particular, that Types I and III are associated with parameter hysteresis as the asymptotic dynamics jump discontinuously between impacting and non-impacting attractors, where the former involve appreciable impulses. In contrast, in Type II, although the previously stable non-impacting periodic trajectory loses stability at grazing, the post-grazing attractor is still local in state space and impacts occur with small relative velocity.

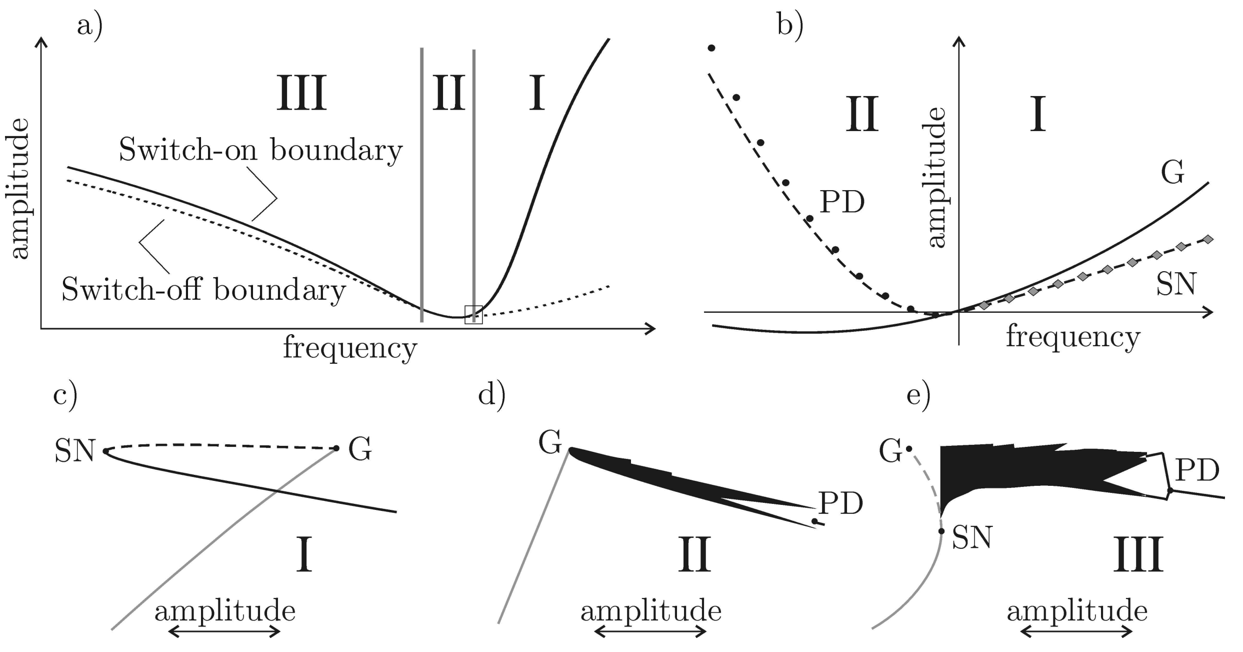

Figure 2. Transition scenarios between non-impacting to impacting asymptotic dynamics under variations in driving amplitude and frequency (G=grazing contact, SN=saddle-node bifurcation, PD=period-doubling bifurcation). In a), the solid line corresponds to the onset of impacting motion under increases in amplitude, while the dashed line corresponds to the termination of impacting motions under decreases in amplitude. Panel b) shows a blow-up of the rectangular region in a) at the boundary between Type I and Type II transitions illustrating the close agreement between the analytical theory (curves) using discontinuity mappings and numerical data (circles and squares). Panels c), d), and e) exhibit the bifurcation scenarios associated with the switching between impacting motions (black) and non-impacting motions (gray) that occurs under variations in driving amplitude. Here, solid curves correspond to stable periodic motions and dashed curves to unstable periodic motions. The black regions correspond to impacting chaotic attractors.

DISCONTINUITY MAPPINGS

The local dynamics in the vicinity of a grazing trajctory for parameter values near the critical parameter values can be analyzed through the introduction of a discontinuity mapping that i) captures the local dynamics in the vicinity of the grazing contact including variations in time-of-flight to the discontinuity and the impact mapping; ii) can be entirely characterized by conditions at the grazing contact; iii) is nonsmooth in the deviation from the point of grazing contact; and iv) can be studied to arbitrary order of accuracy [6]. In this paper, we formulate local mappings in the vicinity of the grazing trajectory for each of the three transition scenarios. We also show how it is necessary to include higher-order terms in the local mapping to accurately capture the bifurcations that occur in an open neighborhood of the co-dimension-two points at the boundaries between the different transition scenarios. As an example, Figure 2b) shows the prediction of the local mapping on the form of the bifurcation curves in the vicinity of a grazing trajectory at the boundary between the Type I and Type II scenarios.

ACKNOWLEDGEMENTS

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