Bubble wall and bubble pair interaction using potential flow theory

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Abstract

In this study the motion and interaction of a bubble with a vertical wall is analyzed in the high Reynolds and low Weber number limit. On this dual limit the inertial effects dominate, and bubbles remain spherical. There are experiments that confirm that this approach is valid. The velocity of the fluid was obtained by expanding the potential in a series of spherical harmonics. The motion equations were obtained using an energy approach.

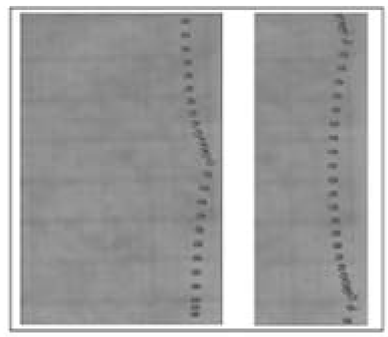

Figure 1: Photographs of experiments of a bubble moving near a wall. The bubble positions for different instants were superimposed into the same plate to show the bubble trajectory

1 Introduction

We study the motion and interaction of a bubble with a vertical wall in the high Reynolds number limit. We consider small Weber number, for which the deformation of the bubble is negligible. The motion of a pair of bubbles has been investigated by several author [3, 1]. In this study, Laplace's equation is solved in terms of spherical harmonics with a method similar to that used by [1]. In figure (1) photographs of experiments are shown. Bubbles moving near a wall, for the same regime as that considered in the theory, move to the wall and bounce repeatedly as they move upwards. We want to analyze this effect with potential flow theory.

2 Theoretical framework

We have to solve the problem of two spheres in motion with velocity U_1 and U_2, moving in a nearly inviscid fluid. The velocity is decomposed in two components, one mean velocity V=U1+U2, and a difference of velocities component W=W1-W2. Laplace's equation must be solved:

$$\nabla^2 \phi = 0 \quad (1)$$

The potential can be expanded in a double series of harmonical spherics centered in each sphere:

$$\Phi_{V_k} = \sum_{i=1}^{n} V_k \cdot a_1 \{ g^1_{mn} \left(\frac{R}{r_1}\right)^{n+1} Y^k_n(\cos\theta_1, \mu) \quad (2)$$

$$+ g^2_{mn} \left(\frac{R}{r_2}\right)^{n+1} Y^k_n(\cos\theta_1, \mu) \} \quad (3)$$

The next step was to calculate the multipole coefficients using the boundary conditions. This was done equating the normal velocity of the fluid in the sphere surface with the normal velocity of the spheres. The coefficients g_{mn} and f_{mn} were obtained in terms of a powers series of the rate radius-bubble separation (R/s).

$$g_m n = 1/2 \sum_{p=0}^{\infty} K_{mnp}(R/s)^p \qquad (4)$$

3 Equations of motion for two bubbles

Equations of motion can be obtained using an energy method, in which the kinetic energy of the system is calculated. As the energy of the system of spheres and fluid in motion will not change, the equations of motion can be obtained with Lagrange equations.

$$T = 1/2 \rho_l V a_{ij} u_i u_j \qquad (5)$$

Thus the kinetic energy is dependent of the velocities Uk and Wk. The system can be transformed to a system with two absolute velocities : x' and y', and two reltive velocities between the bubbles s' and θ'. Motions equation were find by appliying Lagrange equations to the system of particles and the fluid:

$$\frac{d}{dt}[\frac{\partial T}{\partial \dot{q}^i}] - \frac{\partial T}{\partial q^i} = F_i \qquad (6)$$

4 Results

The motion equations obtained in the last section were solved numerically using a Runge Kutta scheme. For our simulation we selected a fixed radius R, and an initial distance to the wall, s= 4R. With the viscosity and g acceleration of gravity, we calculated a terminal velocity $U = gR^2/9\nu$. Simulations were conducted for a wide range of bubble radii and initial bubble-to-wall distances. In figure (2) two typical trajectories are shown. These are in good qualitative agreement with our experiment observations. A complete recount of results can be found in [4].

When we simulate without drag, then the initial distance of the bubble to the wall is recovered and also the velocity of the bubble recovers its initial value. When including a drag in the bubbles,

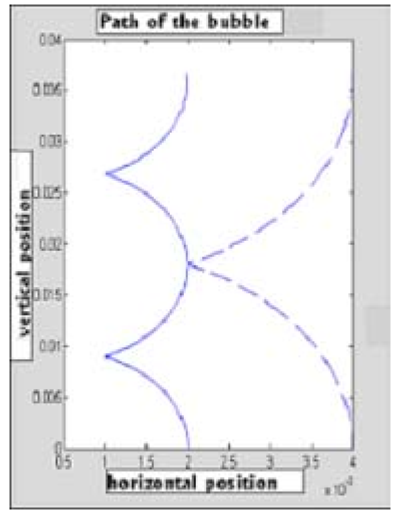

Figure 2: Numerical solution of equation (6) showing the path of the bubble interacting with a wall. Two typical cases, for different radii, are shown.

the motion of the bubbles was damped in each rebound. This work was based on a master Thesis en (IIM,UNAM)[4].

References

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