

## H<sub>∞</sub> CONTROL FOR SMART MULTISTOREY BUILDING STRUCTURES

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**Summary** A dynamic model for multistorey buildings under external excitations is presented. Structured uncertainties are considered to reflect the errors between the model and the reality. H<sub>∞</sub> optimal control for the active control structure is implemented. The H<sub>∞</sub> norm of the transfer function from the disturbances to the errors is minimized. Numerical techniques, which have been done with the help of MATLAB routines, are applied to solve the arising structural control problem.

### INTRODUCTION

The vibrations of flexible structures, like tall buildings, during the earthquake or gust wind may lead to damages and destructions of the structure systems. The control of the building's structural vibrations is an important goal for the structural engineer. Most of the active control systems built at present are mainly aimed at the response reduction to strong wind or quite moderate earthquake excitation. One of the important problems in achieving reliable active control systems that could ensure the safety for strong earthquake is their robustness. Robust control focuses on the issues of the performance and the stability in the presence of uncertainty, both in the parameters of the system and in exogenous inputs. H<sub>∞</sub> optimal robust control technique is used in this paper for vibration suppression of building structures.

### PROBLEM STATEMENT

A dynamic model for multistorey building equipped with actuators under earthquake and wind excitations is presented in this paper. The structure is modeled by linear system using the finite element method. The equations of motion are

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = Hu(t) + F(t) \quad (1)$$

where the vector  $X(t)$  describes the displacements,  $u(t)$  contains the control forces applied by the actuators and  $F(t)$  is the external loading vector.  $M$ ,  $C$  and  $K$  are mass, damping and stiffness matrices.  $H$  is a distribution matrix defining the locations of the actuators. All matrices with appropriate dimensions are constant and real.

As the quality of the model depends on how closely its responses match those of the true plant, structured uncertainties are considered to reflect the errors between the model and the reality. Suppose that the physical parameters  $M$ ,  $C$ , and  $K$  are not known exactly. The nominal matrices  $M^* = mI$ ,  $C^* = cI$ ,  $K^* = kI$  are diagonal. The corresponding perturbations  $\Delta_M$ ,  $\Delta_C$ ,  $\Delta_K$  are unknown but restricted  $\Delta_M = \delta_M I$ ,  $\Delta_C = \delta_C I$ ,  $\Delta_K = \delta_K I$ , ( $-1 \leq \delta_M, \delta_C, \delta_K \leq 1$ ). The actual  $M$ ,  $C$ ,  $K$  are within  $p_M$ ,  $p_C$ , and  $p_K$  percentages of the nominal  $M^*$ ,  $C^*$ ,  $K^*$

$$\begin{aligned} M &= M^* (I + p_M \Delta_M) = m (1 + p_M \delta_M) I \\ C &= C^* (I + p_C \Delta_C) = c (1 + p_C \delta_C) I \\ K &= K^* (I + p_K \Delta_K) = k (1 + p_K \delta_K) I \end{aligned} \quad (2)$$

Let  $X(t)$ ,  $\dot{X}(t)$ , and  $F(t)$  are the inputs of the system and  $y = X(t)$  is the output. To represent the model as linear fractional transformation (LFT) of the natural uncertainty parameters  $\delta_M$ ,  $\delta_C$ ,  $\delta_K$  we isolate the uncertainty perturbations  $\Delta_M$ ,  $\Delta_C$ ,  $\Delta_K$  and denote their inputs as  $y_m$ ,  $y_k$ ,  $y_c$  and their outputs as  $u_m$ ,  $u_c$ ,  $u_k$ . Then the augmented system's form is obtained as follows

$$[\dot{x} \ y_\Delta \ y]^T = G_n [x \ u_\Delta \ u]^T, \quad u_\Delta = \Delta y_\Delta \quad (3)$$

where  $x = [X \ \dot{X}]^T$ ,  $y_\Delta = [y_m \ y_k \ y_c]^T$ ,  $u_\Delta = [u_m \ u_c \ u_k]^T$ . The plant transfer matrix  $G_n$  depends only on the nominal parameters of the system and is known. The uncertainty matrix  $\Delta$  is a block diagonal structural matrix and influences on input/output connection between  $u$  and  $y$  in a way that can be represented as feedback by upper LFT  $y = L_U(G_n, \Delta)u$ .

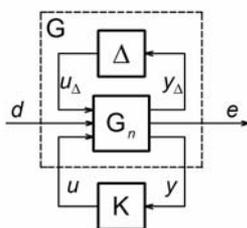


Figure 1.

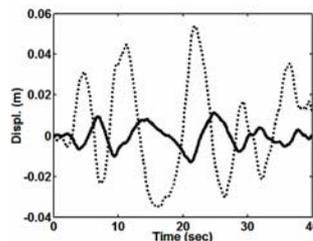


Figure 2.

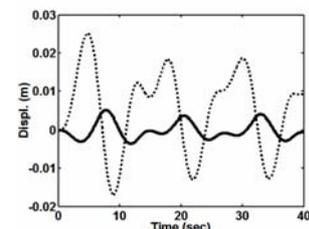


Figure 3.

The linear system (3) can be described with the structural scheme in Figure 1. The uncertainty block  $\Delta$  is supposed to be stable and norm bounded  $\|\Delta\|_\infty \leq 1$ . The block  $K$  is the controller. The nominal plant  $G_n$  has three sets of inputs: uncertainty inputs  $u_\Delta$ , external disturbances  $d$  and control commands  $u$ . Three sets of outputs are generated: uncertainty

outputs  $y_\Delta$ , errors  $e$  and measurements  $y$ . Let  $w = [u_\Delta \ d]^T$  are all external inputs coming to the system and  $z = [y_\Delta \ e]^T$  are all signals characterizing system's behavior. Then the system (3) is transformed in

$$\begin{bmatrix} z \\ y \end{bmatrix}^T = \mathbf{G} \begin{bmatrix} w \\ u \end{bmatrix}^T, \quad u = \mathbf{K}(s)y \quad (4)$$

The transfer matrix  $\mathbf{G}$  is obtained from the nominal model  $\mathbf{G}_n$  and contains weights for the uncertainty, which depends on the control design. The closed loop system's transfer matrix from  $w$  to  $z$  is given by a lower LFT  $z = \mathbf{F}_L(\mathbf{G}, \mathbf{K}) w$ .

The problem of the control design consists in determination of a controller  $\mathbf{K}$  that ensures internal stability of the system (4) keeping the transfer matrix  $\mathbf{F}_L(\mathbf{G}, \mathbf{K})$  between  $w$  and  $z$  minimal in the sense of  $\mathbf{H}_\infty$  norm.

$$\|\mathbf{F}_L(\mathbf{G}, \mathbf{K})\|_\infty = \max_{\omega} \{\bar{\sigma}(\mathbf{F}_L(\mathbf{G}, \mathbf{K})(j\omega))\} \rightarrow \min \quad (5)$$

In the frequency domain the minimization of the  $\mathbf{H}_\infty$  norm minimizes the maximal value of the maximal singular value of  $\mathbf{F}_L(\mathbf{G}(j\omega), \mathbf{K}(j\omega))$ . In the time domain minimizing this norm interpreted as the induced 2-norm gets the minimization of the worst case.

$$\min \|\mathbf{F}_L(\mathbf{G}, \mathbf{K})\|_\infty = \min \left\{ \max_{\|w(t)\|_2 \neq 0} \frac{\|z(t)\|_2}{\|w(t)\|_2} \right\}, \quad \text{where } \|z(t)\|_2 = \sqrt{\int_0^\infty |z_i(t)|^2 dt} \quad (6)$$

## PERFORMANCE AND STABILITY

The robust performance and the robust stability for the closed loop system are of paramount importance. The parameter perturbations can amplify significantly the effect of the external influences. As a result the performance of the closed loop system can be deteriorated before losing the stability. To arise a desirable performance it is necessary that

$$\|\mathbf{W}_p(\mathbf{I} + \mathbf{G}\mathbf{K})^{-1}\|_\infty < 1 \quad (7)$$

for all frequencies. The weight matrix  $\mathbf{W}_p$ , reflects the relative importance of different frequency ranges where the performance is requested.

For robust stability we are interested in finding the smallest perturbation  $\Delta$  ( $\|\Delta\|_\infty < 1$ ) in the sense of  $\bar{\sigma}(\Delta)$  such that destabilizes the closed loop framework. The loop is well-posed and internally stable if and only if the structured singular value  $\mu_\Delta$  is less than one for all frequencies

$$\mu_\Delta = \frac{1}{\min\{\bar{\sigma}(\Delta) : \|\Delta\|_\infty < 1\}} < 1 \quad (8)$$

The quantity  $(\max \mu_\Delta)^{-1}$  is a stock of stability with respect to the structured uncertainty influenced the system.

## NUMERICAL SIMULATIONS

Relevant numerical techniques, which have been done with the help of MATLAB routines, are applied to solve the arising structural control problem and to find the optimal controller. An upper and a lower bound of  $\mu_\Delta$  are calculated. The conclusions concerning the robust stability are made in terms of these bounds. The  $\mathbf{H}_\infty$  norm of the closed loop system with  $\mathbf{H}_\infty$  optimal controller  $\mathbf{K}$  is obtained in request bounds. The respective transfer matrices are in norm less than one. Therefore, the closed loop system achieves nominal and robust performance. Numerical results show high robust performance.

For numerical simulations of the structure's behavior two types of dynamic loading are considered: random white noise modeling an earthquake ground motion and periodic sinusoidal pressure modeling a horizontal wind loading. The responses of the open-loop of the closed-loop systems are compared based on the reduction of the magnitude of the maximum horizontal displacement. Results for the tip of the uncontrolled (dot) and controlled (solid) building structure due to earthquake and wind loadings are presented in Figure 2 and Figure 3 respectively.

## CONCLUSIONS

Robust control design of an uncertain multistorey building structure is considered. Structured uncertainties are introduced. The vector of active control forces subjected to  $\mathbf{H}_\infty$  performance criterion and satisfying the system's dynamic equations such that to reduce the adverse earthquake and wind excitations is determined. High robust performance and robust stability are achieved. A two-dimensional finite element model is utilized for numerical experiments. Comparisons of the controlled and uncontrolled systems demonstrate the effectiveness of the proposed optimal control law.

## References

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