# WEIGHT FUNCTIONS FOR CRACKS IN PIEZOELECTRICS

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*Summary* Two types of weight functions for cracks in piezoelectrics are derived yielding stress and electric displacement intensity factors for point loads/charges and local displacements/electric potentials, respectively. The weight functions are applied to calculate field intensity factors and displacement/electric potential fields in a cracked body under arbitrary electromechanical loading conditions. Further applications include the investigation of effects like bridging or small scale switching on the crack tip loading.

Fracture mechanics of piezoelectric and ferroelectric materials is of increasing interest for applicants of smart materials. The analytical framework of piezoelectric fracture mechanics has been established in the early 90s by researchers like Sosa [1], Pak [2] or Park and Sun [3] giving rise to an increasing interest of scientists in this field. Anyhow, the concept of crack weight functions, well known in fracture mechanics of classical materials since 1970 (Bueckner [4]) has only recently been extended to piezoelectric materials by Ma and Chen [5] as well as McMeeking and Ricoeur [6]. In [5] the derivation of the weight function is based on a work-conjugate integral following Bueckner, in [6] the derivation goes back on a procedure published by Rice [7] leading to a more transparent formulation.

The weight function in fracture mechanics relates the stress intensity factor at the tip of a crack in an elastic body to a point load at an arbitrary location. For a piezoelectric material, this definition is extended to include the effect of point charges and the presence of an electric displacement intensity factor at the crack tip. The weight function for cracks in piezoelectric materials is formulated from Maxwell relationships among the energy release rate with displacements and the electric potential as dependent variables and the applied loads and electric charges as independent variables. Applying the principle of linear superposition and Bettis theorem of reciprocity, one type of weight functions is derived for the different crack opening Modes from any known mixed-mode solution in terms of displacements and electric potentials of the cracked body under specific electromechanical loads. Fig. 1 shows all four field intensity factors for a Griffith crack in PZT-5H due to a near-tip unit line charge at an angle θ to the crack tip and at a distance r=1mm from the crack tip.

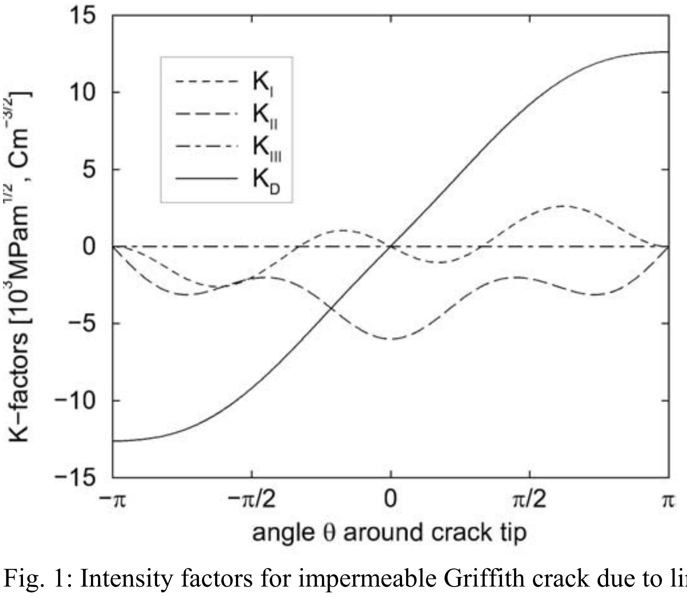


Fig. 1: Intensity factors for impermeable Griffith crack due to line charge at r=1mm and angle θ to crack tip (θ=0 on the ligament)

Another type of piezoelectric crack weight function yielding stress and electric displacement intensity factors due to local mechanical displacements and electric potentials can be derived following a similar principle. Here, the weight functions for the different crack opening modes are calculated from any mixed-mode solution in terms of stresses and electric charge densities. Having once derived both types of weight functions for a specific piezoelectric body containing a crack, field intensity factors can be calculated for arbitrary mixed – Dirichlet and Neumann – boundary conditions. Moreover it is possible to calculate mechanical displacement and electric potential fields in the cracked body knowing the weight functions for the different crack opening Modes and the stress and electric displacement intensity factors for the chosen electromechanical loading, which is demonstrated by examples.

Further results are presented for the Griffith crack taking into account different electric boundary conditions on the crack faces. From the piezoelectric asymptotic solution crack tip weight functions are derived. In connection with a process zone model for ferroelectrics (Ricoeur and Kuna [8]) they are applied to quantify the effect of small scale domain switching close to the crack tip showing a toughening effect if an external electric field is applied parallel to the poling direction. Moreover, weight functions have been used to investigate the sensitivity of numerical errors occurring close to the crack tip on the calculation of the electric displacement intensity factor for electrically permeable cracks (Wippler *et al*. [9]). Finally, the effect of bridging tractions and charges on the field intensity factors is calculated by means of piezoelectric weight functions.

## References

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