XXI ICTAM, 15–21 August 2004, Warsaw, Poland

LARGE-SCALE SEMI-ORGANIZED STRUCTURES IN GEOPHYSICAL TURBULENT CONVECTION

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Summary A new mean-field theory of turbulent convection is developed. In a shear-free turbulent convection the theory predicts the convective wind instability which causes formation of large-scale semi-organized fluid motions in the form of cells. The theory predicts also the convective-shear instability in a sheared turbulent convection which results in appearance of large-scale semi-organized convective rolls. This instability can cause also a generation of convective-shear waves which have a nonzero hydrodynamic helicity. The increase of shear promotes excitation of the convective-shear instability. Predictions of this theory are in a good agreement with the modern knowledge about the atmospheric convective boundary layer and observed semi-organized structures.

In the last decades it has been recognized that the very high Rayleigh number atmospheric convective boundary layer (CBL) has more complex nature than might be reckoned. Besides the fully organized component naturally considered as the mean flow and the chaotic small-scale fluctuations, one more type of motion has been discovered, namely, long-lived large-scale structures, which are neither turbulent nor deterministic [1,2]. These semi-organized structures considerably enhance the vertical transport and render it essentially non-local in nature. In the atmospheric shear-free convection, the structures represent three-dimensional Benard-type cells (cloud cells) composed of narrow uprising plumes and wide downdraughts. They embrace the entire convective boundary layer ($\sim$ 2 km in height) and include pronounced large-scale ($\sim$ 3 km in diameter) convergent flow patterns close to the surface [1,2]. In sheared convection, the structures represent CBL-scale rolls (cloud streets) stretched along the mean wind. Life-times of the semi-organized structures are much larger than the turbulent time scales [1,2]. Thus, these structures can be treated as comparatively stable, quasi-stationary motions, playing the same role with respect to a small-scale turbulence as the mean flow.

In spite of a number of studies, the nature of large-scale semi-organized structures is poorly understood. The Rayleigh numbers, Ra, based on the molecular transport coefficients are very large (of the order of $10^{11} - 10^{13}$). This corresponds to fully developed turbulent convection in atmospheric flows. At the same time the effective Rayleigh numbers, based on the turbulent transport coefficients are not high, e.g., they are less than the critical Rayleigh numbers required for the excitation of large-scale convection. Therefore, the emergence of large-scale structures which are observed in the atmospheric flows seems puzzling.

The main goal of this study is to suggest a mechanism for formation of large-scale semi-organized structures. Traditional theoretical models of the boundary-layer turbulence, such as the Kolmogorov-type closures and similarity theories (e.g., the Monin-Obukhov surface-layer similarity theory) imply two assumptions: (i) Turbulent flows can be decomposed into two components of principally different nature: fully organized (mean-flow) and fully turbulent flows. (ii) Turbulent fluxes are uniquely determined by the local mean gradients. For example, the turbulent flux of entropy (or potential temperature) is given by $\langle s\mathbf{u}\rangle = -\kappa_T \boldsymbol{\nabla}\bar{S}$, where κ_T is the turbulent thermal conductivity, $\bar{S}$ is the mean entropy, $\mathbf{u}$ and s are fluctuations of the velocity and entropy, respectively. However, the mean velocity gradients can affect the turbulent flux of entropy. The reason is that additional essentially anisotropic velocity fluctuations can be generated by tangling of the mean-velocity gradients with the Kolmogorov-type turbulence. The source of energy of this "tangling turbulence" is the energy of the Kolmogorov turbulence. The tangling turbulence is strongly anisotropic and has a steeper spectrum ($\propto k^{-7/3}$) than a Kolmogorov turbulence. The anisotropic velocity fluctuations of tangling turbulence were studied in the first by Lumley (1967). It is shown in this study that the tangling turbulence can cause formation of semi-organized structures due to the excitation of large-scale instability. In particular, the tangling turbulence contributes to the turbulent flux of entropy. Calculations based on the Navier-Stokes equation and the entropy evolution equation formulated in the Boussinesq approximation yield the following expression for the turbulent flux of entropy $\boldsymbol{\Phi} \equiv \langle s\mathbf{u}\rangle$:

$$\boldsymbol{\Phi} = \boldsymbol{\Phi}^* - (\tau_0/6)[5\alpha(\boldsymbol{\nabla}\cdot\bar{\mathbf{U}}_\perp)\boldsymbol{\Phi}^*_\parallel - (\alpha+3/2)(\bar{\boldsymbol{\omega}}\times\boldsymbol{\Phi}^*_\parallel) - 3(\bar{\boldsymbol{\omega}}_\parallel\times\boldsymbol{\Phi}^*)] \, , \qquad (1)$$

where $\bar{\mathbf{U}} = \bar{\mathbf{U}}_\perp + \bar{U}_z\mathbf{e}$ is the mean velocity, $\mathbf{e}$ is the vertical unit vector, $\bar{\boldsymbol{\omega}} = \boldsymbol{\nabla}\times\bar{\mathbf{U}}$ is the mean vorticity, $\bar{\boldsymbol{\omega}}_\parallel = \bar{\omega}_z\mathbf{e}$ is the vertical mean vorticity, τ_0 is the correlation time of the Kolmogorov turbulence corresponding to the maximum scale of turbulent motions, $\boldsymbol{\Phi}^*$ is the turbulent flux of entropy which is caused by the contribution of the Kolmogorov turbulence and it is independent of the mean velocity gradients $\nabla_i\bar{U}_j$, $\boldsymbol{\Phi}^*_\parallel = \Phi^*_z\mathbf{e} = -\kappa_T\boldsymbol{\nabla}_z\bar{S}$, $\alpha = (1+4\xi)/(1+\xi/3)$ is the degree of thermal anisotropy of the background turbulent convection (without mean-velocity gradients), $\xi = (l_\perp/l_z)^{2/3} - 1$. The turbulent flux of entropy can be obtained even from simple symmetry reasoning. Here $l_\perp$ and l_z are the horizontal and vertical scales in which the correlation function $\Phi_z^{(0)}(\mathbf{r}) = \langle s(\mathbf{x})\,u_z(\mathbf{x}+\mathbf{r})\rangle$ tends to zero. In the isotropic case, $l_\perp = l_z$ the parameter $\xi = 0$ and $\alpha = 1$. For $l_\perp \ll l_z$ the parameter $\xi = -1$ and $\alpha = -9/2$. The maximum value $\xi_{\max}$ of the parameter ξ is $\xi_{\max} = 2/3$ for $\alpha = 3$. Thus, for $\alpha < 1$ the thermal structures have the form of columns or thermal jets $(l_\perp < l_z)$, and for $\alpha > 1$ there exist the ''pancake" thermal structures $(l_\perp > l_z)$ in the background turbulent convection.

Now let us discuss the mechanisms of formation of semi-organized structures by a large-scale instability. The mechanism of the convective wind instability, associated with the second term $\boldsymbol{\Phi} \propto -\tau_0\alpha(\boldsymbol{\nabla}\cdot\bar{\mathbf{U}}_\perp)\boldsymbol{\Phi}^*_\parallel$ in the expression for the turbulent flux of entropy [see Eq. (1)], in the shear-free turbulent convection at $\alpha > 0$ is as follows. Perturbations of the vertical mean velocity $\bar{U}_z$ with $\partial\bar{U}_z/\partial z > 0$ have negative divergence of the horizontal velocity , i.e., $\mathrm{div}\,\bar{\mathbf{U}}_\perp < 0$ (provided that $\mathrm{div}\,\bar{\mathbf{U}} \approx 0$). This results in the vertical turbulent flux of entropy and causes an increase of the mean entropy. On the other hand, the increase of the mean entropy increases the buoyancy force and results in the increase of the vertical velocity $\bar{U}_z$ and excitation of the large-scale instability. Similar phenomenon occurs in the regions with $\partial\bar{U}_z/\partial z < 0$ whereby $\mathrm{div}\,\bar{\mathbf{U}}_\perp > 0$. This causes a downward flux of the entropy and the decrease of the mean entropy. The latter enhances the downward flow and results in the instability which also causes formation of a large-scale semi-organized convective cell structure (convective wind). Thus, nonzero $\mathrm{div}\,\bar{\mathbf{U}}_\perp$ causes redistribution of the vertical turbulent flux of entropy and formation of regions with large vertical fluxes of entropy. Thereby the regions with

$\mathrm{div}\,\bar{\mathbf{U}}_\perp < 0$ are separated by the regions with low vertical flux of entropy with $\mathrm{div}\,\bar{\mathbf{U}}_\perp > 0$. This results in a formation of a large-scale circulation of the velocity field. Another mechanism of the convective wind instability is associated with the third term [proportional to $(\alpha + 3/2)(\bar{\boldsymbol{\omega}} \times \boldsymbol{\Phi}^*_\parallel)$] in the expression (1) for the turbulent flux of entropy when $\alpha < -3/2$. This term describes the horizontal flux of the mean entropy. The latter results in increase (decrease) of the mean entropy in the regions with upward (downward) fluid flows. On the other hand, the increase of the mean entropy causes the increase of the buoyancy force, the mean vertical velocity $\bar{U}_z$ and the mean vorticity $\bar{\boldsymbol{\omega}}$. Hence the large-scale convective wind instability is excited. The second term in the turbulent flux of entropy at $\alpha < -3/2$ causes a decrease of the growth rate of the instability because, when $\alpha < -3/2$, it implies a downward turbulent flux of entropy in the upward flow. This decreases both, the mean entropy and the buoyancy force. Note that, when $\alpha < -3/2$, the thermal structure of the background turbulence has the form of a thermal column. Our analysis, based on the linearized momentum equation and the equation for the entropy with the derived expression for the turbulent flux of entropy (1), showed that the growth rate $\gamma_{\rm inst}$ of the convective wind instability of long-wave perturbations $[\beta \equiv (l_0 K)^{-2} \gg 1]$ is given by $\gamma_{\rm inst} \propto \nu_T K^2 \sqrt{\beta}\,|\sin\theta|\,[\alpha - 3/8 - (5\alpha/4)\sin^2\theta]^{1/2}$, where θ is the angle between $\mathbf{e}$ and the large-scale wave vector $\mathbf{K}$ of small perturbations, ν_T is the turbulent viscosity and l_0 is the maximum scale of turbulent motions. We considered here an isentropic basic reference state. For large β the growth rate of the instability is proportional to the wave number K (i.e., $\gamma_{\rm inst} \propto K u_0$) and the instability occurs when $\alpha(5\cos^2\theta - 1) > 3/2$. Here u_0 is the characteristic turbulent velocity in the maximum scale of turbulent motions. Thus there are two ranges for the excitation of the instability, i.e., the first range is for $\alpha_* < \alpha < 3$ and the second range is for $-9/2 < \alpha < \alpha_*$, where $\alpha_* = 3/[2(5\cos^2\theta - 1)]$, and we took into account that the parameter α varies in the interval $-9/2 < \alpha < 3$. The first range for the instability corresponds to the angles $3/10 \leq \cos^2\theta \leq 1$ (the aspect ratio $0 < L_z/L_\perp < 1.5$), and the second range for the instability corresponds to the angles $0 \leq \cos^2\theta < 2/15$ (the aspect ratio $2.6 < L_z/L_\perp < \infty$), where $L_z/L_\perp \equiv K_\perp/K_z = \tan\theta$. Our analysis showed that the maximum growth rate of the instability is attained at the scale of perturbations $L_m \approx 10\, l_0$, and the characteristic time of excitation of this instability is of the order of $(20-30)\tau_0$. Thus the typical length and time scales of the convective-wind motions are much larger than the turbulence scales. This justifies separation of scales which is required for the description of the semi-organized structures in terms of a mean flow.

In a sheared turbulent convection the mechanism of the convective-shear instability is associated with the last term in the expression (1) for the turbulent flux of entropy $[\boldsymbol{\Phi} \propto \tau_0(\bar{\boldsymbol{\omega}}_\parallel \times \boldsymbol{\Phi}^*)]$. Here $\boldsymbol{\Phi}^* = \Phi^*_z \mathbf{e} - \tau_0 \Phi^*_z (d\bar{\mathbf{U}}^{(0)}(z)/dz)$, the second term in the expression for $\boldsymbol{\Phi}^*$ describes the counter-wind heat flux and $\bar{\mathbf{U}}^{(0)}(z)$ is the imposed horizontal large-scale flow velocity (e.g., a wind velocity). The mechanism of the convective-shear instability is as follows. The vorticity perturbations generate perturbations of entropy. Indeed, for two vortices with opposite directions of the vorticity $\bar{\boldsymbol{\omega}}_\parallel$, the turbulent flux of entropy is directed towards the boundary between the vortices. The latter increases the mean entropy between the vortices. Such redistribution of the mean entropy causes increase of the buoyancy force and formation of upward flows between the vortices. These vertical flows generate vorticity. Thus the convective-shear instability is excited. The growth rate of the instability of long-wave perturbations is given by $\gamma_{\rm inst} \simeq \nu_T K^2 (\beta\lambda\sin^2\theta)^{2/3}$, where we considered turbulent convection with a linear shear $\bar{\mathbf{U}}^{(0)} = (\lambda/\tau_0)\, z\, \mathbf{e}_y$ and a nonzero vertical flux of entropy $\boldsymbol{\Phi} = \Phi^*_z\,\mathbf{e}$. Here λ is a dimensionless parameter which characterizes the shear. This instability causes formation of large-scale semi-organized fluid motions in the form of rolls align along the imposed mean velocity $\bar{\mathbf{U}}^{(0)}$. The instability can also result in generation of the convective-shear waves with the frequency $\Omega \simeq \sqrt{3}\nu_T K^2 (\beta\lambda\sin^2\theta)^{2/3}$. These convective-shear waves propagate perpendicular to convective rolls. This finding is in agreement with observations in the atmospheric convective boundary layer, whereby the waves propagating perpendicular to cloud streets have been detected [3]. Remarkably, that the flow in the convective-shear wave has a nonzero hydrodynamic helicity. For perturbations with $K_x = 0$ the convective-shear instability does nor occur. However, for the perturbations with $K_x = 0$, the convective wind instability can be excited, and it is not accompanied by the generation of the convective-shear waves. Our analysis showed that there are two ranges for the excitation of the instability. However, even a small shear causes an overlapping of these two ranges, and the increase of shear (λ) promotes the convective-shear instability with the growth rate $\gamma_{\rm inst} \propto K^{2/3}$, and the frequency of the generated convective-shear waves is $\Omega \propto K^{2/3}$.

Now let us compare the obtained results with the properties of semi-organized structures observed in the atmospheric convective boundary layer. The semi-organized structures are observed in the form of rolls (cloud streets) or three-dimensional convective cells (cloud cells). Rolls usually align along or at angles of up to 10° with the mean horizontal wind of the convective layer, their lengths vary from 20 to 200 km, the widths from 2 to 10 km, and convective depths from 2 to 3 km [1,2]. The typical value of the aspect ratio $L_z/L_\perp \approx 0.14 - 1$. The ratio of the minimal size of the structure to the maximum scale of turbulent motions $L/l_0 = 10 - 100$. The characteristic life time of rolls varies from 1 to 72 hours. Rolls may occur over water surface or land surfaces. The suggested theory predicts the following parameters of the convective rolls: the aspect ratio $L_z/L_\perp$ ranges from very small to 1, and $L/l_0 = 10 - 100$. The characteristic time of formation of the rolls $\sim \tau_0/\gamma_{\rm inst}$ varies from 1 to 3 hours. The life time of the convective rolls is determined by a nonlinear evolution of the convective-shear instability. The latter is a subject of a separate ongoing study. Convective cells may be divided into two types: open and closed [1]. Open-cell circulation has downward motion and clear sky in the cell center, surrounded by cloud associated with upward motion. Closed cells have the opposite circulation. Both types of cells have diameters ranging from 10 to 40 km and aspect ratios $L_z/L_\perp \approx 0.05 - 1$, and both occur in a convective layer with a depth of about 1 to 3 km [1,2]. The ratio of the minimum size of the structure to maximum scale of turbulent motions $L/l_0 = 5 - 20$. The developed theory predicts the following parameters of the convective cells: the aspect ratio $L_z/L_\perp$ ranges from very small to 1, and $L/l_0 = 5 - 15$. The characteristic time of formation of the convective cells $\sim \tau_0/\gamma_{\rm inst}$ varies from 1 to 3 hours. Therefore the predictions of the developed theory are in a good agreement with observations of the semi-organized structures in the atmospheric convective boundary layer.

References

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