NONLINEAR EFFECTS, OBSERVED IN THE PROCESS OF THE LIQUID FLOWING OUT OF THE VIBRATING VESSELS: THEORY, EXPERIMENT AND APPLICATIONS

I.I. Blekhman, L.I. Blekhman, L.A. Vaisberg, V.B. Vasilkov, K.S. Yakimova
Institute for Problems of Mechanical Engineering and "Mekhanobr-Tekhnika" Corp., Saint Petersburg, Russia

Summary A general physical explanation and theoretical description of two nonlinear phenomena observed in the process of the liquid flowing out of the vibrating vessels are proposed. The results of the experimental investigations and of possible applications to the processes of screening flotation are presented.

EXTENDED SUMMARY

The presentation is devoted to two peculiar nonlinear phenomena. The first of them, called vibro-jet effect, has been known for a comparatively long time [1]. It consists in the fact that when a plate with conic holes vibrates in a fluid slow flows of fluid appear in the direction of the narrowing of the holes. The vibro-jet effect is successfully used in a number of technical devices [1-3]. Along with that, this phenomenon was a possible cause of some aviation catastrophes when due to vibration, the fuel stopped coming from the tanks; i.e., there was a vibration closing of the holes. In this case the pressure, facilitating the discharge of the fuel, was balanced by the counter-pressure, appearing due to vibration. The second phenomenon - the vibrational injection of a gas into the fluid - has been discovered quite recently [4]. It consists in the fact that gas is being sucked into the vessel with the fluid through the hole in the lower part of the vessel, vibrating in the gas. Though injection will also take place in the case when the vessel vibrates in the fluid; i.e., we may speak here about a phenomenon of vibrational injection of fluid into fluid.
The theory of vibro-jet effect was considered in [3] and in the book [1]. Here we propose a general explanation comprising, the theory of both phenomena; and also give the results of the experiments.

THE MAIN THEORETICAL RESULTS

The vessel (Fig.1) contains fluid 1, and outside there is either fluid or gas 2. The vessel can be either open or closed, and performs vertical oscillations according to the law $y = -A\sin\omega t$, where A is the oscillation amplitude and ω is the oscillation frequency. The hole is at the bottom of the vessel.
We assume that the well-known formula for hydraulics $Q = \mu F\sqrt{2\Delta P/\rho}$ for discharge is also valid for the instantaneous discharge under the action of pressure difference $|\Delta P|$, variable in time (μ is the coefficient of discharge, ρ is the density of the medium, F is the area of the hole). As a result we obtain the following expressions for the period $T = 2\pi/\omega$ the volume discharge per second of the medium flowing out (Q_+^T, positive) and flowing into the vessel (Q_-^T, negative).

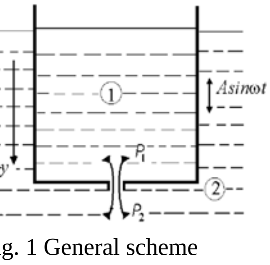

Fig. 1 General scheme of the system

$$Q_\pm^T = \pm\frac{1}{2\pi}\mu_\pm F\sqrt{2\Delta P/\rho_\pm}\,J_\pm(w), \tag{1}$$

where

$$J_\pm(w) = 4\sqrt{2w}\left[\mathbf{E}(k_\pm) - (1-k_\pm^2)\mathbf{K}(k_\pm)\right], \qquad (w \ge 1),$$
$$J_+(w) = 4\sqrt{1+w}\,\mathbf{E}(1/k_+), \qquad J_-(w) = 0, \qquad (0 \le w \le 1), \tag{2}$$

$k_\pm = \sqrt{(w\pm 1)/(2w)}$, $\mathbf{K}$ and $\mathbf{E}$ are the complete elliptic integrals of the first and the second type respectively, $w = \rho_1 hA\omega^2/\Delta P$ is the so-called overload parameter, h is the height of the column of water in the vessel, $\rho_1 = \rho_+$, $\rho_2 = \rho_-$ are the densities of the media 1 and 2 respectively (Fig. 1), $\Delta P = P_1 - P_2$ is the static pressure difference at the hole.
In the case of *vibro-jet effect* the fluids inside and outside of the vessel are the same, so that $\rho_+ = \rho_- = \rho$, $w > 1$, and the total average discharge is

$$Q^T = Q_+^T + Q_-^T = \frac{1}{2\pi}F\sqrt{2\Delta P/\rho}\left[\mu_+ J_+(w) - \mu_- J_-(w)\right]. \tag{3}$$

When the inequality $\mu_+/\mu_- < J_-(w)/J_+(w)$ is satisfied, then in spite of the positive static pressure difference $\Delta P = P_1 - P_2 > 0$, the fluid flows into the vessel (for the period more fluid flowing in than flowing out), and when $\mu_+/\mu_- = J_-(w)/J_+(w)$ then the *effect of vibro-closing the vessel*, described above, takes place. At $w > 5$ the condition of the appearance of the inverse flow can be presented approximately as $\mu_+/\mu_- < 1 - 2.27/w$, and the condition of the vibrational closing as $\mu_+/\mu_- = 1 - 2.27/w$.

In the case of the *vibrational injection of gas into the fluid* the vessel is believed to be open and the pressure of gas on the surface of the fluid and at the outlet from the hole is believed to be the same. Then $\Delta P = \rho_1 gh$ and formula (1) for the average for the period T gas and liquid discharges $Q_+^T = Q_f^T$ and $Q_-^T = Q_g^T$ will be presented in the form

$$Q_+^T = Q_f^T = \frac{1}{2\pi}\mu_f F\sqrt{2gh}J_+(w), \quad Q_-^T = Q_g^T = -\frac{1}{2\pi}\mu_g F\sqrt{2gh\rho_f/\rho_g}J_-(w) \tag{4}$$

Here, unlike (1) the values referring to the fluid are marked with symbol “ f ”, while those referring to the gas are marked with the symbol “ g ” and $w = A\omega^2/g$.

THE RESULTS OF THE EXPERIMENT

The experiments on studying the vibrational injection were made on a special vibrational stand 157 A-YC elaborated at the "Mekhanobr" Institute. On the counter of that stand an open glass cylindrical vessel was fixed, it was 300 mm high with the inner diameter 58 mm. In the center of the bottom of the vessel there was a round hole. A certain level of water was kept in the vessel. The vessel was endowed with vertical vibration whose frequency and amplitude might be changed. The process of vibrational injection was observed in the stroboscopic light. That helped to establish that during every oscillation period one bubble of air was being sucked into the vessel and one drop of water was flowing out. Fig. 2 shows a photo of a vibrating vessel with the air being intensively sucked into it.

In the process of the experiment they measured the dependence of average volume discharge of water $Q_{f\,\exp}^T$, flowing out of the hole on the parameter w with the fixed oscillation amplitude $A = 2.5$ mm, of the height of the column of water in the vessel $h = 200$ mm and the diameter of the hole $d = 2{,}6$ mm.

The experiments have shown that a sufficiently intensive sucking-in of the air into a vessel, observed in the conditions of the experiment, begins at the value $w \approx 2.5$, while, according to the above-presented theory, it must begin at w somewhat more than 1. This can be explained by the effect of the surface tension when bubbles are being formed.

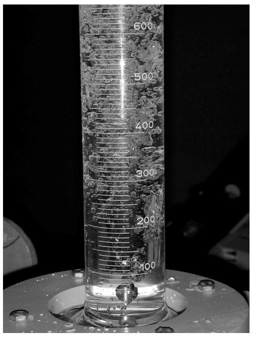

Fig. 2. Vibrational injection of gas into fluid

According to the results obtained, we performed the comparison of the coefficient $\mu_f^{T_+}$ with the value μ for the case of the stationary flow of the fluid through the hole. In this case $\mu_f^{T_+}$ implies the value of the coefficient μ_f, calculated from the experimental average discharge $Q_f^{T_+} = Q_f^T T/T_+$ for that part T_+ of the period T when the fluid is flowing out of the vessel. It was found that the values $\mu_f^{T_+}$ at $w > 4$ do not differ much more from the value μ which, it seems, speaks in favor of the elements of the theory stated above.

To apply the phenomenon of the vibrational injection to the processes of screening and flotation a series of experiments were carried out with the vessel, which had in its bottom a system of holes (round or slot-shaped) instead of a single hole. It was established that even at the area of the holes about 50% of the area of the bottom the process runs with the same intensity as with a single hole if the intensity of vibration is increased by about 10%.

CONCLUSION

It seems that the phenomena considered, as well as many other remarkable effects, appearing under the action of vibration on the gas-fluid and similar systems, are worthy of the attention of specialists. They may find important applications in technique, in particular, in chemical industry and in mineral processing.

The work is carried out with the support of the Russian Foundation of the Fundamental Researches (Grant SSC 1521.2003.8) and the Saint Petersburg Scientific Center.

REFERENCES

[1] Vibration in technology in 6 volumes: Handbook, **4**, Mashinostroenie, Moscow, 1978-1981 (in Russian).
[2] Blekhman I.I. :Vibrational Mechanic – Nonlinear Dynamic Effects, General Approach, Applications. World Scientific 2000. 509 p.
[3] Blekhman I. I., Vaisberg L. A., Korovnikov A. N. Investigations of the processes, machines and apparatus for classifying the material according to size. *Interdepartmental Proceedings, Mekhanobr, Leningrad*. 1988. p. 35-46 (in Russian).
[4] Blekhman I.I., Blekhman L.I., Vaisberg L.A., Vasilkov V.B., Yakimova K.S. In collection *Scientific discoveries, Moscow, Russian Academy of Natural Sciences*. 2002. p. 60-61 (in Russian).
[5] Blekhman I.I., Blekhman L.I., Vaisberg L.A., Vasilkov V.B., Yakimova K.S. Nonlinear effects, observed in the process of the liquid flowing out of the vibrating vessels. *Doklady Acad. of Sc., Russia.* 2003. **391**(2). p. 185-188.