TOPOLOGICAL CHAOS IN SIMPLE MIXERS

Matthew D. Finn*, Stephen M. Cox**, Helen M. Byrne*
*School of Mathematical Sciences, University of Nottingham, Nottingham NG7 2RD, UK
**School of Applied Mathematics, University of Adelaide, Adelaide 5005, Australia

Summary Topological chaos is discussed in a two-dimensional batch mixer and a three-dimensional static mixer, under very viscous flow conditions. Topological ideas may be used to calculate a minimum stretch rate for certain flows, but cannot predict the size of the region in which this stretching is achieved. Numerical simulations of dye advection are used to test whether topological ideas can be used to practically enhance mixing quality. In two dimensions it is found that material stretch rates are in tight accordance with theoretically predicted values dependent on flow topology. Furthermore, effective mixing is readily achieved in a useful sized domain. However, in three dimensions we find flow features which make topological arguments less practical for improving mixing quality.

INTRODUCTION

A mixing regime commonly studied in the last twenty years, and exploited in industries such as polymer manufacturing, is slow viscous flow, or Stokes flow, in which simple stretching and folding mechanisms are employed to generate chaotic particle paths [1, 2]. Although chaotic advection is readily created, optimising mixing parameters for a given purpose is extremely difficult because chaotic flow simulations are computationally expensive and usually prohibit exhaustive parameter-space searches. A further stumbling block for many optimisation procedures is that mixing performance can depend sensitively on geometrical parameters (e.g. the size of a stirring impeller), and on the fluid properties. However, very recently, an elegant idea has emerged which potentially allows one particular mixing measure, material stretch rate, to be robustly built-in to a mixer. The concept is Topological Chaos, as described in a remarkable theoretical and experimental paper by Boyland, Aref and Stremler [3]. Topologically chaotic flows are a subset of chaotic flows in which the flow topology alone is responsible for achieving a guaranteed minimum theoretical material stretch rate, in some part of the domain, regardless of the exact geometry or fluid details (viscosity, compressibility, rheology, etc). The topological approach, however, while widely applicable, cannot predict the size of the region in which this stretch rate is achieved. A careful investigation is needed into whether topological ideas can generate good mixing *in practice*.

STIRRING IN TWO DIMENSIONS

A requirement for topologically chaotic flow is that a batch mixing device must contain at least three moving stirrers [3]. Surprisingly, therefore, very few industrial batch mixers are equipped to exploit topological chaos. Those that are appear to employ it almost accidentally, as they are not marketed as possessing any special topological flow properties. Almost all mixing devices which have so far been modelled mathematically are incapable of producing topological chaos. However, such flows are readily generated in a two-dimensional model batch stirring device (BSD) [3, 4], which is the subject of our study. The BSD is a generalisation of the translating, rotating mixer [5], consisting of a circular cylinder in which the fluid is stirred by an arbitrary number of moving stirring rods. In order to perform meaningful simulations of the chaotic flow, accurate expressions for the velocity field are needed because numerical errors grow exponentially in time. Although we cannot determine an exact solution for the BSD velocity, we have developed a computationally cheap, spectrally accurate, series expression for the streamfunction, which allows high-precision simulations of tracer advection experiments. Some sample advection simulation results are shown in Figure 1.

A key question we have addressed is whether topological ideas may be useful in practice for guiding mixer design, paying particular attention to whether stretch rates predicted by the topological theory are achieved in a physically useful area of the mixing domain [6]. Our form of solution may be used to calculate accurately other physical quantities of interest, such as the energy input required for mixing, allowing studies of mixing efficiency. The effect of stirrer shape on mixing quality is also discussed, because stirrers of ‘paddle’ shape cross-section, for instance, might be better, intuitively, than the mathematically simpler circular cross-sections. Non-circular stirrers may be considered by using appropriate conformal transformations in the series solution for the velocity field. Although the change in stirrer cross-section has no influence on the flow topology, it may have an impact on energy usage, or be used to enhance stretching rate above that predicted by the topological theory, allowing a more economical, or a higher performance mixer, respectively.

STIRRING IN THREE DIMENSIONS

Although batch mixers are used in industry, continuous-throughput mixing in three dimensions is more widely used. An extra spatial dimension to the flow means that static mixing is possible, i.e. no time dependence is required in the velocity field in order to achieve chaotic particle advection [7]. Time-dependence commonly is employed, but many successful patented static mixer designs exist (e.g. by Kenics, Mixtec, TAH Industries and Sulzer–Chemtech). These static mixers generally consist of a circular cylindrical pipe containing a judiciously designed insert that causes fluid in the cylinder to be ‘cut and folded’ in the cross-section as it is pumped axially. We study a static mixer called the braided pipe mixer (BPM) inspired by the two-dimensional BSD [8]. The BPM consists of a circular cylindrical pipe containing smaller

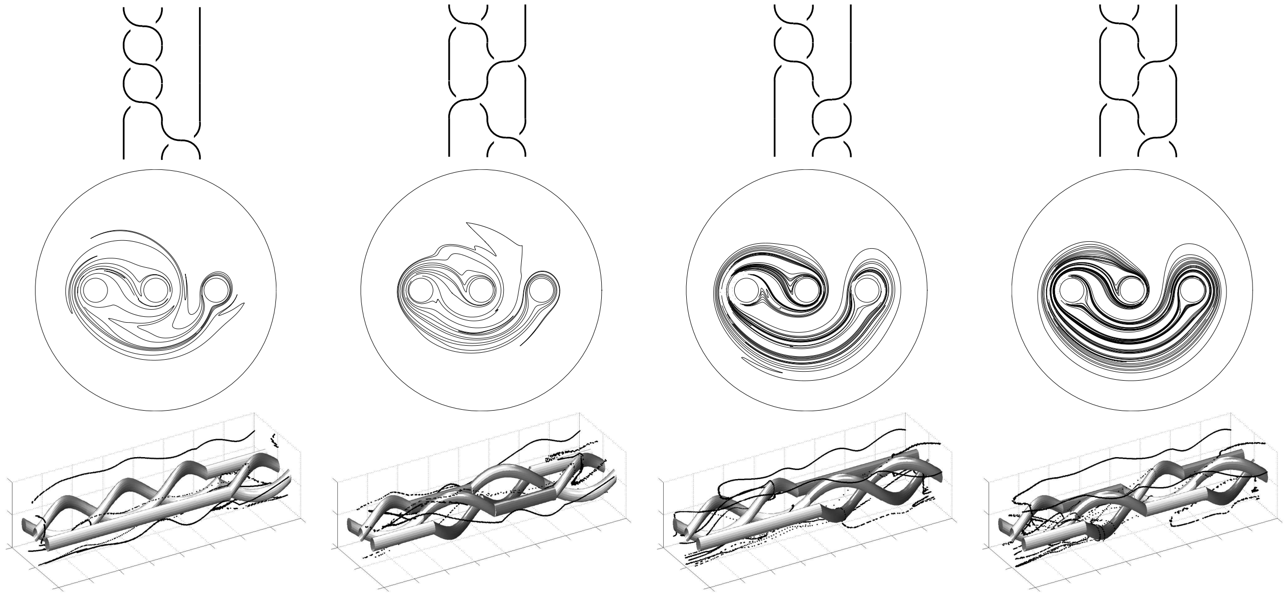

Figure 1. Example results of dye advection in the BSD (middle row) and BPM (bottom row), with stirring regimes corresponding to different braid topologies (top row). In the BSD the flow topology refers to how the stirring rods are moved around each other; in the BPM the topology describes how the internal pipes are intertwined. The images show the fate of a straight line initial dye blob after a 'moderate' stirring time.

twisted pipes that cause the fluid to mix as it is driven along by an imposed pressure gradient. In such static mixers the axial coordinate occupies the role played by time in the two-dimensional BSD. The correspondence is not exact, however, because different fluid particles travel at different axial speeds, and some even come to rest on the internal pipes. The velocity field for the BPM is constructed using a series approach, as with the BSD. Sample BPM dye advection results are shown in Figure 1 (the outer pipe is not shown). It was recently suggested that the obstacles in the pipe could be braided to generate effective chaotic mixing in three dimensions [3, 9]. Furthermore, it was conjectured that these pipes could be reduced in thickness and still generate good mixing. By direct comparison with the two-dimensional work, we discuss whether topological ideas for two-dimensional flows may be applied to improve mixing in steady three-dimensional flows.

CONCLUSIONS

The two-dimensional BSD, containing three or more stirring rods, is capable of generating topologically chaotic flow. We provide numerical evidence to support the observation of Boyland *et al.* [3] that the region of high stretch is comparable with that through which the stirring elements move during operation of the device. Calculated material stretch rates were found to be in close agreement with the corresponding theoretical predictions. The theoretical stretch rates were achieved in the BSD even as the radii of the stirring rods were reduced, although the stretching became more localised around the trajectories of the rods. In the three-dimensional BPM we found that it was difficult to distinguish between the mixing performance of different braid topologies according to material stretch rate and measures based on the axial distribution of tracer, in contrast with the BSD where braid performances were clearly ordered and in agreement with topological theory. Although one might intuitively expect static mixer design could be improved using topological considerations, there is no formal topological chaos theory for three-dimensional flows. By considering particle advection in an artificial two-dimensional flow using the axial coordinate as 'time', we find that topological chaos *is* present in the BPM; however, in this system, the regions of high material stretch rate corresponded exactly with those regions where particle advection along the device was very slow using the real time. On a practical time scale the fast stretching is not seen, and topological chaos appears to offer no significant benefit to the mixer design.

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