NONLINEAR DYNAMICS OF PINNED-PINNED CYLINDERS IN AXIAL FLOW

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Summary It is known that cylinders with supported ends subjected to high enough axial flow develop divergence and at higher flow coupled-mode flutter, as shown experimentally and confirmed by linear theory. Also, the same dynamics is predicted by linear theory for the closely related problem of a pipe conveying fluid, but post-divergence flutter in this case has never been observed. Its non-existence was confirmed by nonlinear theory. The problem of the cylinder in flow is re-examined in this paper by means of weakly nonlinear theory. It is shown that post-divergence flutter does exist, but not as an instability of the trivial equilibrium, but as a Hopf bifurcation emanating from the divergence solution. For high enough flow, interesting dynamics follow, including quasiperiodicity and chaos. Reasons for the different dynamics with internal and external flow are explored.

BACKGROUND

It has been shown, in the 1960s, that cylinders with supported ends immersed in sufficiently high axial flow are subject to divergence (buckling), and at higher flow to coupled-mode flutter – by means of linear theory, confirmed by experiment [1,2]. Linear theory predicts the same dynamical behaviour for the closely related system of a pipe conveying fluid with supported ends (i.e. a cylinder with internal flow): divergence followed by flutter. In this case, however, flutter was never observed. This was confirmed and elucidated by nonlinear theory [3,4]. The question is “why does the same not hold true for external flow?”

To answer this question, the problem of a cylinder with simply supported ends (“pinned-pinned”) immersed in axial flow is re-examined by nonlinear theory, as outlined in what follows.

A WEAKLY NONLINEAR MODEL

Assuming the lateral deflection of the cylinder to be $v \sim O(\varepsilon)$, and consequently the axial extension to be $u \sim O(\varepsilon^2)$, and presuming that the cylinder centreline is extensible, the equations of motion for a pinned-pinned cylinder in confined axial flow have been derived, correct to $O(\varepsilon^3)$. These equations are based on the potential flow derivation of the lift on a flexible slender body derived by Lighthill [5], modified by including semi-empirical expressions for the viscous forces and, to deal with vertically mounted systems, buoyancy, gravity and pressure-loss related forces as well [6,7]; see also Ref. [8] for cantilevered cylinders. These equations may be expressed as

$$L_i\left(u\left(x,t\right),v\left(x,t\right)\right)=0, \qquad i=1,2,$$

where the L_i are nonlinear differential operators, with $i=1,2$ corresponding, respectively, to the predominantly axial and transverse motion PDEs.

The equations are discretized by Galerkin’s method, leading to a set of coupled ODEs, the solution of which is complicated by the presence of inertial nonlinearities. Solutions are obtained by Houbolt’s finite-difference method and, to a limited extent, using AUTO.

TYPICAL RESULTS

A typical bifurcation diagram is shown in Fig. 1(a). It is seen that this particular system loses stability by divergence at a nondimensional flow velocity $U \simeq \pi$, in conformity with linear theory. Coupled-mode flutter, however, associated with another loss of stability of the trivial equilibrium, as predicted by linear theory, does not arise: the bifurcation leads to an unstable solution. Instead, at approximately the same U as predicted by linear theory, the non-trivial static solution becomes unstable by a Hopf bifurcation, leading to flutter.

Hence, post-divergence flutter does exist, as seen in experiments – unlike the situation with internal flow. The difference has been found to be related to the frictional terms. The dynamics of the pipe problem (internal flow) has been shown to be independent of frictional forces, since they are exactly counterbalanced by the pressure-loss forces along the pipe; as a result, both pressure-loss and frictional forces vanish from the equation of motion [4]. This is not true for external flow, in which the mean pressure is largely unaffected by frictional effects on the cylinder. Some of the frictional terms give rise to terms in the equation of motion of a different derivative form *vis-à-vis* the inviscid-force terms and hence the internal flow case. These are the terms responsible for eliminating post-divergence restabilization, even in the absence of dissipative terms. It is also shown by energy considerations that, for flutter to arise from the trivial equilibrium, even in terms of linear theory, the frictional force coefficients in the longitudinal and lateral directions must be unequal [7]; although this is generally true, the restriction seems to be peculiar. In fact, the flutter obtained by nonlinear theory can arise also when these frictional coefficients are equal, and indeed this is the case in the results shown in Fig. 1(a). Hence, although viscous frictional forces are small and by no means dominant, their effect on the post-divergence behaviour of the system is profound.

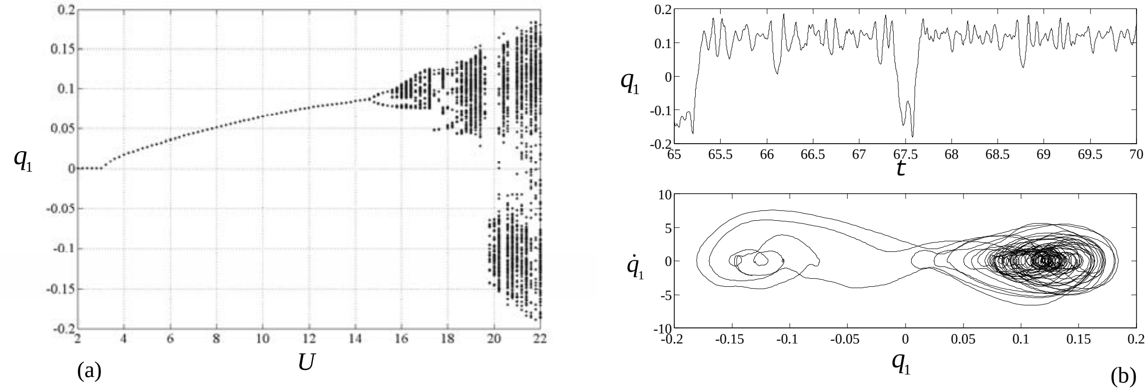

Figure 1. (a) A typical bifurcation diagram; (b) time trace and phase-plane plot for U=21.8

The post-flutter dynamics can be quite interesting, as seen in Fig. 1(a,b). Depending on parameters, the system may develop quasiperiodic and sometimes chaotic behaviour. For the results in Fig. 1(a), the system develops flutter via a Hopf bifurcation at $U \simeq 14.6$. The resulting limit cycle then becomes unstable via a torus bifurcation at $U \simeq 15.7$, indicating that quasiperiodic solutions are possible thereafter. For example at U=17, a time history, and phase-plane and power spectral density plots (none shown here) all indicate a quasiperiodic oscillation.

At higher flow velocities, AUTO reveals the emergence of a number of unstable solutions as the flow is increased; their existence makes chaotic motion more likely. Indeed, in the bifurcation diagram of Fig. 1(a), obtained by the finite difference method (FDM), it is found that chaotic motions are possible over the range $U \simeq 20$ to 22. The time trace and phase-plane plots of Fig. 1(b) show chaotic motions for U=21.8. Similar behaviour has been confirmed to exist over the whole range of $U \simeq 20$ to 22.

CONCLUSIONS

The nonlinear dynamics of a slender cylinder with simply supported ends immersed in axial flow has been explored by means of newly derived weakly nonlinear equations of motion for the case where inextensibility of the centreline is not invoked. It is found that the system loses stability by a pitchfork bifurcation leading to divergence, and the new equilibrium at higher flow becomes unstable by a Hopf bifurcation, leading to flutter. With further increases in the flow velocity, the dynamics becomes more complex, and quasiperiodic and chaotic solutions have been obtained. The different post-divergence dynamics for internal and external flow is discussed and the source of the differences identified.

ACKNOWLEDGEMENT

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