BIFURCATIONS OF NONLINEAR NORMAL MODES OF LINEAR OSCILLATOR WITH STRONGLY NONLINEAD DAMPED ATTACHMENT.

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Summary Damped nonlinear normal modes in a linear oscillator coupled to small damped strongly nonlinear attachment are considered by combining the invariant manifold approach and multiple scales expansion. Three distinct time scales correspond to fast vibrations, evolution of the system towards the nonlinear normal mode (NNM) and time evolution of the invariant manifold. Cusp catastrophe scenario is proved to be the only possible for the invariant manifold in time – amplitude – damping domain.

Introduction

Recently it has been demonstrated that various systems comprised of linear substructures and strongly nonlinear attachments demonstrate localization and irreversible transient transfer (pumping) of energy to prescribed fragments of structure dependent on initial conditions and external forcing [1-4]. Addition of relatively small and spatially localized attachment leads to essential changes in the properties of the whole system. Unlike common linear and weakly nonlinear systems, systems with strongly nonlinear elements are able to react efficiently on the amplitude characteristics of the external forcing in a wide range of frequencies [1,3,4].
The systems under consideration give rise to a new concept of nonlinear energy sink (NES). It was demonstrated [2,4] that the possibility of the energy pumping/resonance capture phenomenon in non-conservative systems can be understood and explained by studying the energy dependence of the non- linear undamped free periodic solutions (non-linear normal modes (NNM,[5-7])) of the corresponding conservative system that is obtained when all damping forces are eliminated. However, that any practical implementation of the energy pumping mechanism requires more detailed understanding of the dynamics of real damped system. Time dependence is crucial feature of the invariant manifolds if the damping is present. Moreover, it will be demonstrated that the topological structure of the invariant manifold is strongly time-dependent and can undergo bifurcations depending on the value of the damping coefficients.

Description of the model and its analysis

Let us consider the following system, which consists of linear oscillator and small strongly nonlinear attachment , described by the set of equations:

$$\ddot{y}_1+\varepsilon(\dot{y}_1-\dot{y}_2)P(y_1-y_2)+y_1+\varepsilon Q(y_1-y_2)=0;\quad \varepsilon\ddot{y}_2+\varepsilon(\dot{y}_2-\dot{y}_1)P(y_2-y_1)+\varepsilon Q(y_2-y_1)=0 \qquad (1)$$

where ε<<1 is a small parameter which establishes the order of magnitude for coupling, damping and mass of the nonlinear attachment. Coupling terms are considered to be symmetric and therefore functions P and Q are presented in a form of even and odd polynomials respectively:

$$P(\xi)=\sum_{j=0}^{n}\lambda_j\xi^{2j},\ \ Q(\xi)=\sum_{j=0}^{m}q_j\xi^{2j+1},\lambda_j,q_j\geq 0 \qquad (2)$$

with at least one of λ_j and q_j is nonzero. Changes of variables $v=(y_1+\varepsilon y_2)/(1+\varepsilon)$, $w=y_1-y_2$, $\delta=1/\chi$ $\chi=\varepsilon^{1/3}$, $V=\chi^{-1}v$, $W=w$, $\varphi_1\exp(it)=\dot{V}+iV$, $\varphi_2\exp(it)=\dot{W}+iW$ reduce system (1) to the following form:

$$\dot{\varphi}_1-\frac{i\chi^2}{2}(\varphi_2-\varphi_2^*\exp(-2it))=0$$

$$\dot{\varphi}_2+\chi(\delta[\frac{i}{2}(\varphi_2-\varphi_2^*\exp(-2it))+\frac{1}{2}(\varphi_2+\varphi_2^*\exp(-2it))P(-\frac{i}{2}(\varphi_2\exp(it)-\varphi_2^*\exp(-it)))+ \qquad (3)$$

$$+\exp(-it)Q(-\frac{i}{2}(\varphi_2\exp(it)-\varphi_2^*\exp(-it)))]-\frac{i}{2}(\varphi_1-\varphi_1^*\exp(-2it)))=0$$

Standard multiple scales expansion with respect to parameter χ demonstrates that in the conditions of 1:1 resonance the dynamical flow is attracted to nonlinear normal mode. The second-order approximation leads to exactly solvable equation for time evolution of the invariant manifold of this NNM ($\tau_2=\chi^2 t$, $\varphi_{10}=R_1\exp(i\gamma_1)$; $\tilde{\varphi}_{20}=R_2\exp(i\gamma_2)$; $\gamma=\gamma_1-\gamma_2$, $Z=R_2^2$) :

$$\frac{\partial Z}{\partial\tau_2}[\{1-2\sum_{j=0}^{m}\frac{q_j}{2^{2j+1}}C_{2j+1}^{j}Z^j\}\{1-2\sum_{j=0}^{m}\frac{q_j}{2^{2j+1}}C_{2j+1}^{j}Z^j-4Z\sum_{j=0}^{m-1}\frac{(j+1)q_{j+1}}{2^{2j+3}}C_{2j+3}^{j+1}Z^j\}+\{\sum_{j=0}^{n}\frac{\lambda_j}{2^{2j}}\frac{(2j)!}{(j!)^2(j+1)}Z^j\}x$$

$$x\{\sum_{j=0}^{n}\frac{\lambda_j}{2^{2j}}\frac{(2j)!}{(j!)^2(j+1)}Z^j+2Z\sum_{j=0}^{n-1}\frac{\lambda_{j+1}}{2^{2j+2}}\frac{(2j+2)!}{((j+1)!)^2(j+2)}Z^j\}]=-\frac{1}{\delta}Z\sum_{j=0}^{n}\frac{\lambda_j}{2^{2j}}\frac{(2j)!}{(j!)^2(j+1)}Z^j \qquad (4)$$

It may be proved that, despite large number of coefficients and complicated structure of (4), the topology of the invariant manifold is surprisingly simple:

1) if $q_0>1$, then $Z(\tau_2)$ decreases monotonously;
2) if $q_0<1$ and at least one of q_j, $j>0$, is positive, then for sufficiently small λ_j there exist two points with divergent derivative $\partial Z/\partial\tau_2$, giving rise to three branches of $Z(\tau_2)$). It is easy to conclude that two of these branches will be stable and one – unstable. As any of λ_j grows, these zeros will eventually disappear by mechanism of trivial cusp catastrophe.

As the topology of the invariant manifold is similar for all possible choices of the coefficients q_j and λ_j (with restrictions formulated above), it is possible to restrict the investigation by the simplest example. Namely, we take $q_1=8$, $q_j=0$ for all $j\neq1$ and $\lambda_0=\lambda$, $\lambda_j=0$ for all $j\neq0$. The shape of the surface $Z(\tau_2,\lambda)$ for $\delta=2.5$ is presented at Fig. 1. In order to illustrate the effect of the bifurcations of the invariant manifold on the dynamics of the system under consideration direct numerical simulation of (1) is performed for the following parameters: $\varepsilon=0.064$, $q_1=8$, $\lambda_0=0.2$ (Fig.2).

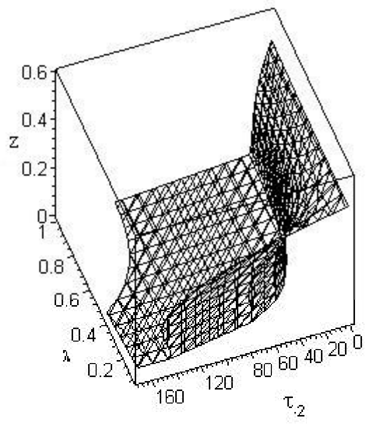

Fig. 1. *Shape of the invariant manifold for the set of parameters $q_1=8$, $\delta=2.5$, $0<\lambda<1$.*

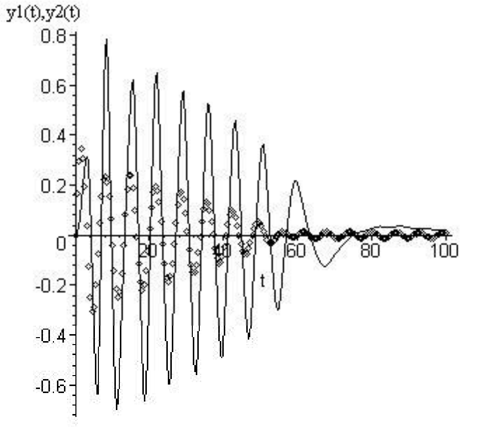

Fig. 2. *Simulation of System (1) for the set of parameters mentioned above, $\lambda=0.2$, $y_1(0)=0$ $dy_1/dt(0)=0.35$, $y_2(0)=0$, $dy_2/dt(0)=0$ (◊◊◊◊◊◊◊ - $y_1(t)$, ———— - $y_2(t)$).*

The picture clearly demonstrates that at about t=20 the trajectory of the system attains the regime of nonlinear normal mode, characterized by simultaneous behavior of $y_1(t)$ and $y_2(t)$. However at about t=50-60 this regime is broken down with rather abrupt decrease of both amplitudes. The phase trajectory of the coupled system completely leaves ("jumps out" from) the resonance manifold and the nonlinear normal mode is totally destroyed as a result of passage through the bifurcation.

Conclusive remarks

We have considered the dynamics of linear oscillator with strongly nonlinear attachment. The above results demonstrate that in the conditions of 1:1 resonance the system under consideration evolutes according to three distinct time scales:

- scale τ_0 corresponds to the fast vibrations of the system with frequency close to unity;
- scale τ_1 corresponds to evolution of the slow variables towards the regime of nonlinear normal mode;
- scale τ_2 corresponds to evolution of the invariant manifold of the NNM.

Time evolution of the invariant manifold may be accompanied by two bifurcations, if the linear frequency of coupling spring is small enough. In this case the bifurcations disappear with increase of the damping coefficient via cusp catastrophe.

Presence of bifurcations of the invariant manifold has rather essential effect on the dynamics of the system. Namely, passage through the bifurcation is able to destroy the regime of nonlinear normal mode and facilitate the energy dissipation. The latter observation leads to certain clue for practical design of the nonlinear energy sinks – the damping coefficient should be chosen in order to ensure the possibility for bifurcations of the NNM invariant manifold. Failure to do so will result in a loss of NES ability to dissipate the energy of vibrations.

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