Non-destructive testing of wood by wave propagation

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Summary In this paper the determination of the material properties of a rectangular wooden bar is presented and the detection of defects in the bar is discussed. The material properties of the bar are evaluated by theoretical and measured dispersion curves using parametric model fitting. A numerical finite difference model with second- and fourth-order approaches is developed. To detect a defect a numerical time-reversal experiment using this model is discussed.

INTRODUCTION

Non-destructive testing of wood is challenging because of its anisotropy and inhomogeneity. It can be tested by ultrasonic waves in high (>1 MHz) [1] or in deep (< 100 kHz) frequency range. In high frequency range the waves are not dispersive but wood has a high damping. A simple "time-of-flight" method can be used on a small cube but it is difficult to test a few meters long bar. Furthermore in this frequency range the wavelength and the natural inhomogeneities, like annual rings, are in the same range, thus they can disturb the measurement.

In low frequency range the damping is weaker and the wavelength is longer but the waves become to be dispersive. A larger bar can be tested and the inhomogeneities do not disturb the measurement but the "time-of-flight" method can not be used. The velocity change caused by dispersion has to be considered for the testing.

PROBLEM STATEMENT

Let us consider a wooden bar with rectangular cross-section. Wood will be modeled as an orthotropic material and the main directions of the orthotropy are parallel to the sides of the bar. The dimensions of the bar are 2700x20x25 mm.

In the first case we are interested in the material properties of the bar. Theoretical and experimental dispersion curves of the bar will be determined and the material properties will be evaluated by parametric model fitting. In the second case we are interested in detecting a defect in the bar. A numerical finite difference model with second- and fourth-order approaches is developed and a numerical time-reversal experiment using this model is discussed [5].

DETERMINATION OF THE MATERIAL PROPERTIES

Theoretical dispersion curves of the bar

A semi-analytical finite element method [2] is applied to determine the dispersion curves of the bar. The cross-section of the bar is discretized by two-dimensional finite elements and in the longitudinal direction harmonic functions were applied. The following displacement functions were used:

$$u(x, y, z, t) = N(x, y)\cos(\omega t + kz)$$
$$v(x, y, z, t) = N(x, y)\cos(\omega t + kz)$$
$$w(x, y, z, t) = N(x, y)\sin(\omega t + kz)$$

where u, v and w are the displacements in x-, y- and z-directions, $N(x,y)$ denotes the biquadratic approximation function, ω the angular frequency, k the wave number and t the time. This leads to an eigenvalue problem:

$$(\mathbf{K}(k_i) - \omega\mathbf{M}(k_i))\mathbf{v} = \mathbf{0}$$

in which **K** denotes the stiffnes-, **M** the mass matrix, and **v** the displacements of the nodes. The eigenvalues of this equation system are the quadrates of the eigenfrequencies ω for the wave number k_i. Solving this eigenvalue problem for different wave numbers k_i results in a dispersion diagram.

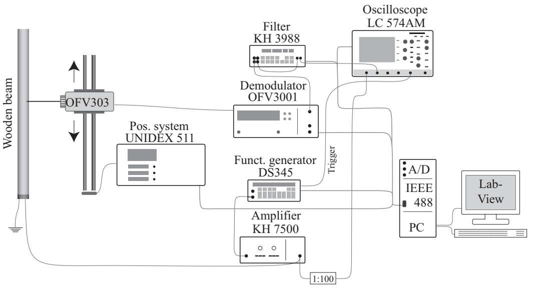

Figure 1. Measurement setup.

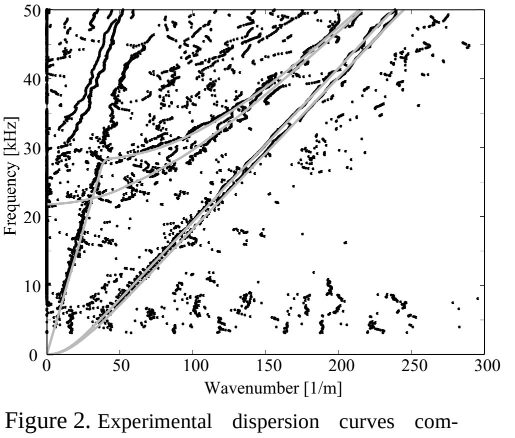

Figure 2. Experimental dispersion curves compared to the theoretical curves evaluated with the determined material properties: C_{22}=1505 N/mm^2, C_{33}=15073 N/mm^2, C_{44}=480 N/mm^2, C_{55}=1350 N/mm^2, C_{66}=1410 N/mm^2.

Evaluation of the dispersion curves from measurements

The measurement setup on Fig. 1. was used to detect the displacements of the bar. The measurement processes are repeated in few hundred points along the longitudinal axis. Using the linear prediction method [4] the dispersion curves of the bar were evaluated from the measurements (black points on Fig. 2).

Parametric model fitting

The total least squares method [1] was applied to evaluate the material properties. The dispersion relation is the relationship between material parameters (Θ) and measurement data $\mathbf{z}(\omega, k)$, in implicit form: $\mathbf{f}(\Theta,\mathbf{z})=0$. In our case there is no analytical relationship, a numerical one was used: $\mathbf{f}(\Theta,\mathbf{z})=\omega_{eig}-\omega_m$, the difference between the calculated (eigenvalue problem) and the measured frequencies. The problem was linearized and solved by the Lagrangian multiplier method [1]. To distinguish between the points of the different wave modes and the points which do not represent a wave mode a statistical test [1] was applied to classify the points into inlier and outlier. The results are presented on Fig. 2.

SIMULATION OF THE WAVE PROPAGATION BY FINITE DIFFERENCES

The applied governing equations are the equations of motion, the orthotropic material's law and the kinematic relations:

$$\sigma_{ij,j} = \rho u_{i,tt}, \qquad \sigma_{ij} = C_{ijkl}\varepsilon_{kl}, \qquad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}).$$

The equations are discretized in space and time domain by second- and fourth-order finite differences on a staggered grid [5]. The second-order finite difference approach of the first derivative of u_x with respect to x and second derivative of it with respect to t are:

$$\frac{\partial u_x}{\partial x} \cong \frac{-{}_{i-1,j,k}u_x^n + {}_{i,j,k}u_x^n}{\Delta x}, \qquad \frac{\partial^2 u_x}{\partial t^2} \cong \frac{{}_{i,j,k}u_x^{n-1} - 2\,{}_{i,j,k}u_x^n + {}_{i,j,k}u_x^{n+1}}{\Delta t^2},$$

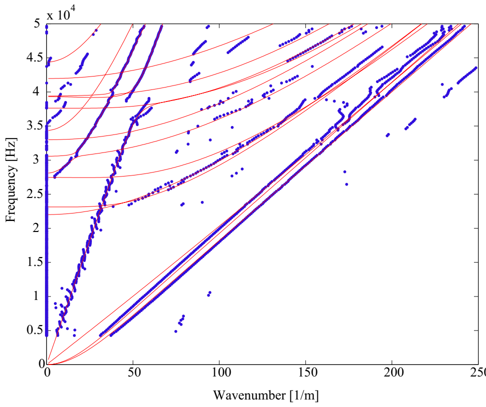

Figure 3. Theoretical dispersion curves determined by the semi-analytical finite element method and experimental ones by simulation (orthotropic bar, second-order approach).

where i, j and k denote the indices of the cell on the grid. The derivatives of u_y and u_z with respect to x, y, z, respectively can be calculated in a similar way. A fourth-order approach was applied in space for the first derivative:

$$\frac{\partial u_x}{\partial x} \cong \frac{{}_{i-2,j,k}u_x^n - 27\,{}_{i-1,j,k}u_x^n + 27\,{}_{i,j,k}u_x^n - {}_{i+1,j,k}u_x^n}{24\Delta x}.$$

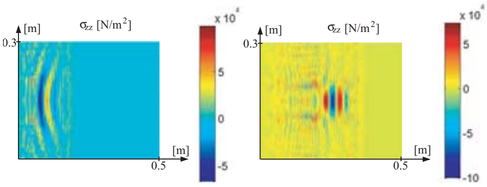

Figure 4. Playing back and focusing of the waves (isotropic plate, second-order approach).

The model was verified by the dispersion curves. In a virtual measurement the displacements of the bar in the simulation were detected. The dispersion curves of the bar were determined by the linear prediction method and compared to the theoretical curves (Fig. 3.).

The finite difference model can be applied in a numerical time-reversal experiment [5] to detect a defect in a structure. By this method structural waves in the bar are excited in an experiment and the displacements are recorded in several points. The recorded signals including the reflections from the defect are played back in the numerical model of the structure, and the waves interfere just at the position of the defect.

A numerical example of this technique is presented in Fig. 4. A notch is implemented in the finite difference model in the middle of an isotropic plate. Longitudinal waves are excited. Waves are detected in a virtual measurement at the end of the plate and than reversed in the time domain and played back in the numerical model without notch (Fig. 4). Limitations of this method should still examined especially the influence of the orthotropy and of the frequency content compared to the geometrical dimensions (slenderness) of the bar.

CONCLUSION

Experimental and theoretical determination of the dispersion curves are presented. It is shown that the material properties of a wooden bar could be determined using the dispersion curves and parametric model fitting. The wave propagation phenomenon was simulated by second- and fourth-order finite differences. A notch was implemented in the model and a numerical time-reversal experiment is considered in the isotropic case.

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