THREE-DIMENSIONAL CORRECTION OF TWO-DIMENSIONAL FRACTURE CRITERIA USING A CONSTRAINT FACTOR

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Summary The three-dimensional (3D) stress state at the crack tip is transformed into quasi-two-dimensional one by introducing the effective elasticity constants governed by the stress or strain constraint factors (SCF). It is shown, that the variation of the deformation state and, as a consequence, of the fracture criteria along the crack front is predicted by the variation of the SCF, while 3D correction may be defined by integrating SCF along the crack front. The proposed approach is illustrated by an example of SENB specimen.

INTRODUCTION

For a long time most of the fracture analyses have been focussed on two-dimensional (2D) approach restricted by two limit cases, plane stress and plane strain, respectively. The deformation state in the vicinity of the crack tip is, however, of three-dimensional (3D) nature, therefore, a great amount of effort, recently, has been spent on the determination of three-dimensional effects [1]. Among the important issues to be clarified is the relationship between 2D and 3D fracture criteria.

In this report the thickness correction of the originally two-dimensional fracture criteria for cracked bodies of the finite thickness used in linear fracture mechanics is considered. The three-dimensional stress state at the crack tip is transformed into quasi-two-dimensional one by introducing the effective elasticity constants governed by the stress or/and strain constraint factors (SCF) [2]. The concept of effective elasticity characteristics suggested in [3] is employed.

CORRECTION CONCEPT

The stress and strain state in the vicinity of the crack tip is almost in the plane strain state, vanishing away from it and approaching the plane stress state. This 3D state is predefined by the dominant out-of-plane resultant. The 3D effect is regarded along the crack front co-ordinate z and may be characterised by stress constraint factor d [2] or modified strain constraint factor k [4], both of them depending on Poisson's ratio ν:

$$d(z)=\frac{1}{\nu}\frac{\sigma_{zz}(z)}{\sigma_{xx}(z)+\sigma_{yy}(z))}, \qquad k(z)=\frac{\nu-1}{\nu}\frac{\varepsilon_{zz}(z)}{\varepsilon_{xx}(z)+\varepsilon_{yy}(z)}. \tag{1}$$

Here, the limit value of d is zero, in case of plane stress, and unity, for plane strain, while the limit value of k is unity, in case of plane stress, and zero, for plane strain. It is necessary to note, that $k + d = 1$.

By applying the methodology used in the elasticity theory and a concept of the position-dependent effective elasticity constants, the 3D state may be transformed into quasi-two-dimensional one. The variation of the state variables along the crack front is, finally, governed by a single variable - constraint factor d or k (1). Originally, the effective elasticity modulus E is simply expressed by the factor d, while using first order Taylor's expansion by factor k, respectively:

$$E_{eff}(z)=E\frac{1}{1-\nu^2 d(z)}, \qquad E_{eff}(z)=E\frac{1}{1-\nu^2}\left(1-\nu^2 k(z)\right). \tag{2}$$

Similar expressions may be derived for Poisson's ratio or Lame constants.

On this basis it easy to show, that the crack front variation of the position-dependent 3D fracture criteria such as stress intensity factor $K(z)$ or energy release rate $G(z)$ is also predicted by the variation of the constraint factor. Assuming that the variation of fracture criteria may be presented as 2D value K_{2D} corresponding to plane strain and 3D correction $\Delta K_{3D}(z)$,

$$K(z)=K_{2D}+\Delta K_{3D}(z). \tag{3}$$

For the fixed front length t, the resultant value is

$$K_{3D}=K_{2D}+\Delta\overline{K}_{3D}\left(K_{2D}-K_{2Dc}\right), \tag{4}$$

where K_{2Dc} is 2D value corresponding to plane stress. The dimensionless correction factor is obtained taking into account the state variation predefined by (2) and by the integration along the crack front t

$$\Delta\overline{K}_{3D}=-\frac{1}{t}\int_t k(z)\mathrm{d}z. \tag{5}$$

Having obtained the variation of the constraint factor $k(z)$, (for example, numerically), the above expressions (3)-(5) may be applied to 3D evaluation and correction of the results of 2D analysis.

NUMERICAL RESULTS

The proposed approach has been applied to the simulation of fracture characteristics of the open-mode three-point bending SENB specimens having different thickness t. In order to recover the 3D fields required for the computation of SCF, the suitable 3D FE element model comprising h-adaptive in-plane and structured through thickness mesh [4] has been proposed. The obtained through the thickness variation of the strain constraint factor $k(\bar{z})$ at the tip of the notch for specimens with different relative thickness $\bar{t}$ is presented in Fig. 1, while the corresponding variation of the relative effective modulus $\overline{E} = E_{eff} / E$ is given in Fig. 2. The relative thickness $\bar{t} = t / h$ presents the ratio of specimen thickness t to its height h.

Finally, the theoretical results are compared to those obtained directly using 3D FE and displacement extrapolation technique. The through the thickness variation of numerically obtained stress intensity factor $\overline{K}(\bar{z}) = \overline{K}(\bar{z}) / K_{2D}$ is presented in Fig. 3. A comparison of the direct numerical simulation and theoretical model (4-5) for different thickness is presented in Fig. 4.

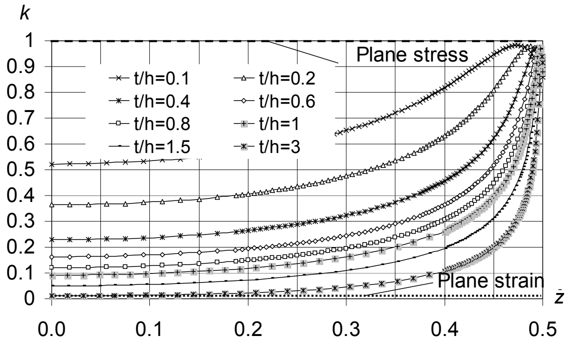

Fig. 1. Through the thickness variation of the strain constraint factor

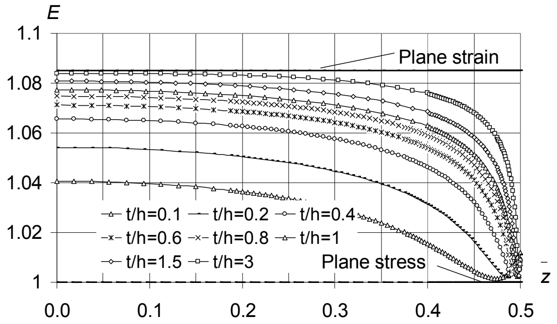

Fig. 2. Through the thickness variation of the relative effective modulus

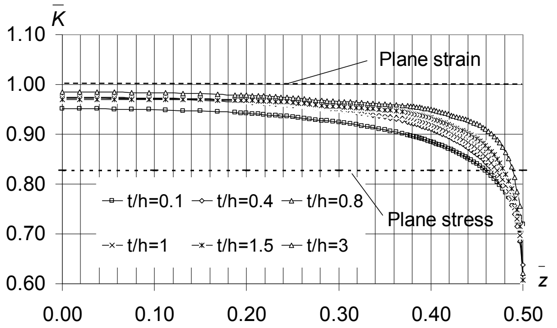

Fig. 3. Through the thickness variation of the relative stress intensity factor

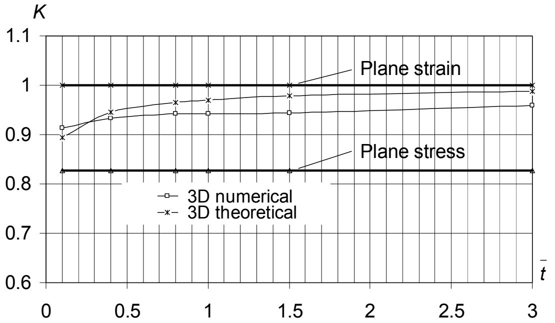

Fig. 4. Variation of the relative stress intensity factor for different thickness

CONCLUSIONS

It is shown, that the actual variation of the fracture criterion along the crack front is predicted by the variation of SCF, while 3D correction is described by the integrated values of SCF. The 2.5% differences occurring between the theoretical expression (5) and direct simulation may be explained by the inaccuracy of the 3D FE models, methodology of computation of fracture criteria and boundary effects on the free surface.

References

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