XXI ICTAM, 15-21 August 2004, Warsaw, Poland

A TWO-DIMENSIONAL ANALYSIS OF SURFACE ACOUSTIC WAVES IN FINITE ANISOTROPIC SOLIDS WITH ELECTRODES

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Summary A two-dimensional theory based on the expansion of displacements in exact solutions from semi-infinite solids offers complete equations and boundary conditions for surface acoustic waves in finite elastic solids similar to well-known plate theories for high frequency vibration applications. By considering the electrode layer and discontinuous electrodes on an elastic substrate, the mass effect is evaluated through the integration of displacements along the thickness direction with simplified equations.

INTRODUCTION

The analysis of surface waves in elastic solids has been clearly demonstrated with semi-infinite solids by assuming displacements decay exponentially along the thickness coordinate, and solutions including the velocity are obtained by applying the traction-free boundary conditions [1]. While surface waves found in some occasions like in earthquakes can be treated like travelling in infinite media, in other applications such as electronic devices utilizing surface acoustic waves (SAW) for frequency control have to be considered as in bounded elastic or piezoelectric solids requiring precise evaluation of complication factors like the periodic electrodes known as interdigital transducers.

It is known that the wave propagation in bounded elastic solids is difficult to analyze because of the reflections in the boundaries produce many modes and their overtones, making both deformation and vibration frequency have many strong couplings. One typical method for the analysis of bulk acoustic waves in bounded solids is Mindlin plate theory [2], which uses series expansion of the displacements to reduce the three-dimensional elasticity equations. Mindlin plate theory and its variations like Lee plate theory [3] have been widely used in the high frequency vibration analysis of quartz crystal plates for their applications in piezoelectric resonators [4], and subsequent applications include the finite element method which enhanced their practical capabilities for complicated structures [5, 6, 7]. For surface acoustic waves in bounded solids, a similar two-dimensional theory is proposed by Wang and Hashimoto [8] with the displacements expanded exponentially with exact surface acoustic wave solutions. It is found that the two-dimensional theory has only two terms for isotropic material and three for anisotropic materials, different from the plate theories that require a truncation procedure to improve the accuracy with limited terms, or the order of the plate theory.

In this study, the periodic electrodes are considered for their mass effect on the SAW velocity, and continuity conditions of displacements are applied so the periodicity can be accounted for. The solutions with finite number of electrodes are obtained from a set of successive boundary conditions and extended to very large number of equations for the limiting case there are infinite number of electrodes. With the periodic solutions for a SAW resonator structure, the effect of the finite sizes of the structure, including the thickness, length, and width of both substrate and electrodes, can be precisely predicted. For demonstration purpose, we analysed a simple device problem with straight-crested waves.

TWO-DIMENSIONAL THEORY FOR SURFACE ACOUSTIC WAVES

It is known that the displacements of surface acoustic waves travelling in x_1 direction in a semi-infinite solid are

$$(u_1, u_2) = (A, B) e^{k\beta x_2} e^{ik(x_1 - ct)}, \tag{1}$$

where $u_1, u_2, A, B, k, \beta, c, t$ are displacement of x_1 direction, displacement of x_2 direction, amplitude of x_1 direction displacement, amplitude of x_2 direction displacement, wavenumber, decaying index, velocity, time, respectively [1]. For a given material, we can find two pairs of real β from the equations of motion. To ensure the exponential decaying of the displacements with negative x_2 in our coordinate system, we must choose positive β_1 and β_2 in (1). Applying traction-free boundary conditions on the surface where $x_2 = 0$, we can find the velocity c with the corresponding β_1 and β_2. Now for a finite solid, we consider the displacements as

$$u_j(x_1, x_2, x_3, t) = \left[u_j^{(1)}(x_1, x_3, t) e^{k\beta_1 x_2} + u_j^{(2)}(x_1, x_3, t) e^{k\beta_2 x_2}\right] e^{ik(x_1 - ct)},\ j = 1,2, \tag{2}$$

where β_1 and β_2 are known and $u_j^{(n)}(j = 1,2; n = 1,2)$ are to be determined. This assumption is natural because we can expect that the solutions of solids with finite thickness or depth can be accurately represented from the semi-infinite solutions. Following the procedure of higher-order plate theory by Mindlin [1], we apply the variational equation

$$\int_{-h}^{0} dx_2 \int_A \left(T_{ij,i} - \rho \ddot{u}_j\right) \delta u_j dA = 0, \tag{3}$$

where $T_{ij}^{(n)}, \rho, h, A$ are stress, density, thickness, and faces, respectively. By integrating (3) with known displacement patterns in (2) over the thickness coordinate, we can write out the equations of motion as

$$\sum_{n=1}^{2}\int_{A}\left(T_{ij,i}^{(n)} - k\beta_n T_{3j}^{(n)} + F_j^{(n)} - \rho\sum_{m=1}^{2} A_{mn}\ddot{u}_j^{(m)}\right)\delta u_j^{(n)} dA = 0. \tag{4}$$

The two-dimensional quantities in (4), $T_{ij}^{(n)}, F_j^{(n)}, A_{mn}$, are defined by the two-dimensional variables in (2) and details are in [8] along with the corresponding boundary conditions. Obviously, (4) can be treated like the plate equations of Mindlin and others for the bulk acoustic wave analysis. Equation (4) represents a set of four two-dimensional equations in terms of displacement components given in (2). The equations and the solution procedure associated with (4) are validated through the precise agreement of results from present two-dimensional theory and three-dimensional equations.

MASS EFFECT OF ELECTRODES AND THEIR PERIODIC STRUCTURE

For a thin electrode layer on top of elastic solids, by neglecting the stiffness of the thin layer, which is much smaller in comparison to the substrate, we have (4) revised as

$$\sum_{n=1}^{2}\int_{A}\left[T_{ij,i}^{(n)} - k\beta_n T_{3j}^{(n)} + F_j^{(n)} - \rho\sum_{m=1}^{2} A_{mn}(1+R)\ddot{u}_j^{(m)}\right]\delta u_j^{(n)} dA = 0, \tag{5}$$

with the mass ratio of the electrode defined as

$$R = \frac{\bar{\rho}\bar{h}}{\rho h}, \tag{6}$$

where $\bar{\rho}, \bar{h}$ are density and thickness of the electrode, respectively. With (5), fully electroded elastic solids can be studied for the effect on the displacements and SAW velocity.

For an elastic solid with periodic electrodes, we can use (4) and (5) alternatively for unelectroded and electroded regions with separated displacement solutions. By applying the continuity conditions to ensure the displacements and stresses are continuous and the boundary conditions are satisfied at the ends, we shall have a set of equations for the periodic structure for displacements and velocity. It is expected that there will be a velocity equation with large number of variables and corresponding matrix, enabling us to evaluate them for both numerical and analytical solutions.

CONCLUSIONS

Utilizing the two-dimensional surface acoustic wave theory, a thin metal layer on top of elastic solids is considered for the mass effect on the displacements and velocity. For a periodic structure with discontinuous electrodes, the equations are alternated between the unelectroded and electroded regions with the continuity boundary conditions, resulting a set of equations for precise consideration. By solving the boundary condition equation numerically and analytically, we have the displacement and velocity solutions of a finite periodic structure along with the mass effect of electrodes.

ACKNOWLEDGMENT

This study is supported by *Qian* River Fellow Fund established jointly by Zhejiang Province and Ningbo University and administered by Ningbo University.

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