Mechanics of elastic and viscous magnetic filaments

A.Cebers, I.Javaitis

Insitute of Physics, University of Latvia, Salaspils-1,LV-2169,Latvia

Elastic magnetic filaments have interesting applications in biotechnology [1,2]. They are encountered also in living world, for example in magnetotactic bacteria [3]. Elastic magnetic filaments have interesting static and dynamic properties which are studied by Kirchhoff model of elastic rod taking into account magnetic forces. The total energy of rod is

$$E = \frac{1}{2}C\int \frac{1}{R^2}dl - \frac{2\pi^2 a^2\chi^2 H_0^2}{\mu+1}\int (\vec{h}\vec{t})^2 dl - \int \Lambda dl \tag{1}$$

Here C is the curvature elasticity constant, a is a radius of rod, χ is its magnetic susceptibility, $\mu = 1 + 4\pi\chi$, R is the radius of curvature of the centerline, $\vec{t}$ is the tangent vector to the centerline, Λ is the tension accounting for inextensibility of the rod. The equations for the rod's tangent angle and tension are obtained if the isotropic Rouse dynamics is assumed $\zeta\vec{v} = \vec{K}$ ($\zeta = \frac{4\pi\eta}{\ln L/a+c}$ is friction coefficient per length unit) and the condition of local inextensibility is imposed. In dimensionless form they read (characteristic length L ($2L$ is the length of the rod), characteristic time $\frac{\zeta L^4}{C}$ and tension $\frac{C}{L^2}$)

$$\theta_t = -(\theta_{lll} + \frac{1}{2}(\theta_l^3)_l) - (\theta_l\Lambda)_l - \Lambda_l\theta_l + Cm(\sin 2\theta)_{ll} - Cm(\theta_l)^2\sin(2\theta) \tag{2}$$

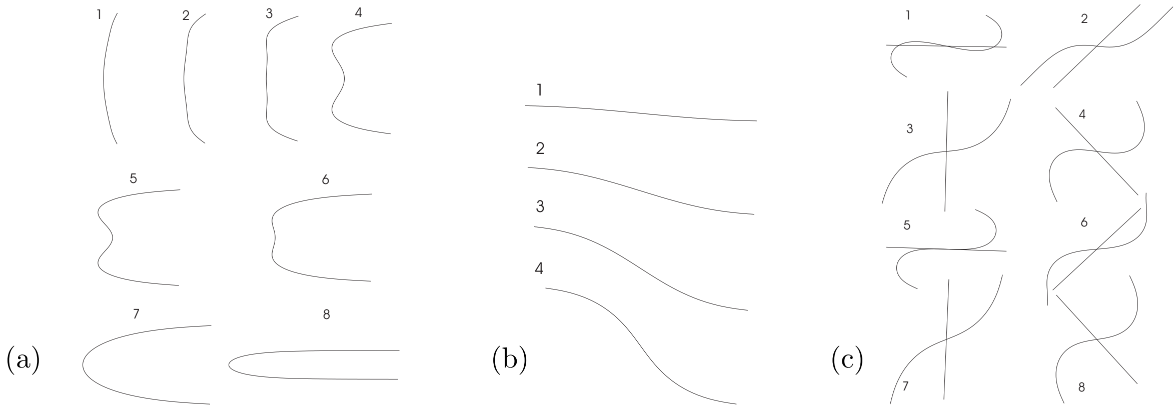
and

$$\theta_l^2\Lambda - \Lambda_{ll} = -\theta_l(\theta_{lll} + \frac{1}{2}\theta_l^3) + Cm\theta_l^2 2\cos 2\theta + Cm(\theta_l \sin(2\theta))_l \tag{3}$$

Here $Cm = \frac{2\pi\chi^2 H_0^2\pi a^2 L^2}{(\mu+1)C}$ is the magneto elastic number characterizing the ratio of the magnetic and elastic forces. Equations (2),(3) describe the behaviour of the elastic magnetic filaments in different situations. Starting from slightly bent configuration of the rod normal to the magnetic field "U" like turns (hairpins) are formed as metastable configurations (Fig.1a). Stable hairpins exist also with nonequal length of their legs. Introducing the phase lag between the direction of the rotating field and the tangent of the rod $\beta = \omega t - \vartheta$ the equation for the angle β is equivalent to the equation (2) with the forcing term $\omega\tau$ on the right side. In this case there is several regimes of the rod dynamics. For low frequencies of the field the rod rotates synchronously with it and has characteristic bent shapes (Fig.1b). Above the critical frequency which is close to the given by relation $(\omega\tau)_c = 2.37Cm$, which is obtained from simplified model, the periodic regime arises with characteristic bendings and straightenings of the rod (Fig.1c). The simplified model of the rod dynamics the equation for the tangent angle of the rod apart from the regularizing term of curvature elasticity is similar to the equation for the phase lag of the elongated viscous magnetic drop under the action of rotating magnetic field [4].

$$\omega\tau = \frac{\partial\beta}{\partial t} - \frac{\partial^2}{\partial l^2}(\frac{1}{Bm}\beta + \sin 2\beta) + \epsilon\frac{\partial^5\beta}{\partial l^4\partial t} \tag{4}$$

Here $\tau = \frac{(\mu+1)\zeta L^2}{2\pi^2\chi^2 H_0^2 a^2}$ is characteristic viscous relaxation time, $Bm = \frac{2\pi\chi^2 H_0^2 a}{(\mu+1)\gamma}$ is the magnetic Bond number characterizing the ratio of magnetic and capillary forces but a small parameter ϵ accounts for the viscous torques arising at the bending of the viscous filament. Since the function $\frac{1}{Bm}\beta + \sin 2\beta$ is nonmonotonous the formation of the shockwave of the tangent occurs at frequencies of the rotating field larger than critical (Fig.1d). From the balance of torques follows that the magnetic torque leads to the normal shearing force $F_n' = -T_0$. Shearing force of such kind arises also in the chains of the magnetic particles in magnetorheological suspensions due to the magnetic dipolar interaction between neighbours. On the other hand the particles are held together due to magnetic interaction forces.



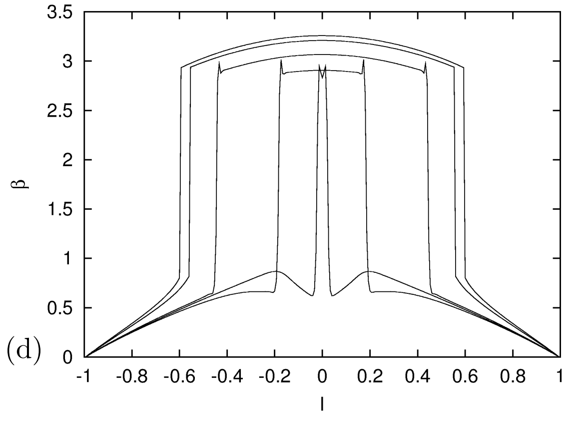


Figure 1: (a) $10^4 t = 0.69(1)$, $3.47(2)$, $6.97(3)$, $17.36(4)$, $34.72(5)$, $52.08(6)$, $69.44(7)$, $1333.33(8)$, $(Cm = 26)$; (b) $\omega\tau = 10(1)$, $30(2)$, $50(3)$, $60.6(4)$, $(Cm = 25)$; (c) $10^2 t = 1.04(1)$, $1.32(2)$, $1.56(3)$, $1.82(4)$, $2.08(5)$, $2.34(6)$, $2.6(7)$, $2.86(8)$, $(Cm = 25,\ \omega\tau = 300)$; (d) $t = 0.28,\ 0.56,\ 1.11,\ 1.67,\ 2.78$ starting from the curve on the center, $(Bm = 1.5;\ \omega\tau = 5.0;\ \epsilon = 10^{-7})$.

The following relation for the number of particles in the chain N at the critical frequency of the chain breaking due to the transition to nonsynchronous regime holds

$$MaN^2 = 2\pi(\frac{3}{2}\sin 2\beta_c + \beta_c)(\ln\frac{L}{a} + c) \tag{5}$$

where $\beta_c = \frac{\pi}{2} - \frac{1}{2}\arccos 1/3$. Relation (5) shows in agreement with experimental data [5] that the number of particles which may be hold together by the magnetic interaction forces depends on Mason number $Ma = \frac{12\eta\omega}{M^2}$ as $N = \frac{const}{Ma^{1/2}}$. This shows that the model of elastic magnetic rod may play the same role in the understanding of the properties of magnetorheological suspensions which the Kirchhoff model plays for the understanding of biopolymers [6].

The model of elastic magnetic rod allows also to understand some phenomena in the magnetotactic bacteria which contains the chains of magnetic particles. For the chain of magnetic particles the curvature elasticity comes from the magnetic interaction between neighbouring particles. In the case of chain of ferromagnetic particles the curvature elasticity constant turns out to be $C_m = \frac{m^2}{2d^3}d$, where m is the magnetic moment of particle but d is its size. The value of the curvature elasticity constant allows to estimate the critical compression force at which the chain buckles $\Lambda_c = \pi^2 K_c/L^2$ which in the case of the magnetite magnetosomes with ten particles of the size $d = 0.1\ \mu$m [3] in the chain gives $\Lambda_c = 12.3\quad pN$. This force is in the range of the forces arising in different biological systems - molecular motors, DNA elasticity [6].

References

1. P.S.Doyle et al., Science, 295 2237 (2002)
2. P.Mayer et al.,Mater.Res.Soc.Symp.Proc., 463 57 (1997)
3. V.Scherbakov et al., Eur.Biophys.J., 26 319 (1997)
4. A.Cebers, Phys.Rev.E, 66 061402 (2002)
5. S.Melle et al., Phys.Rev.E 68 041503 (2003)
6. T.R.Strick et al., Rep.Prog.Phys., 66 1 (2003)