Plastic mass flow of sand under action of pore pressure gradient

Victor N. Nikolaevskiy

The model of poroplasticity has to include the non-associated flow rule, developed (1971) for the Coulomb media with dilatancy in the well-known form. The porous matrix equilibrium is expressed in terms of the effective (Terzaghi) stress σ_{ij} and p - the pore pressure. The Darcy law is sufficient for fluid dynamics description. Let illustrate it for axial symmetry:

$$\frac{\partial \sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{r}-\frac{\partial p}{\partial r}=0 \qquad \phi(w-v) = -\frac{k}{\eta}\frac{\partial p}{\partial r} \qquad (1)$$

Here k - the permeability, and η - the fluid viscosity, ϕ - the porosity; w, v - the "true" radial velocity of the fluid and solid phases. The matrix transfer into plastic flow state if $\sigma_\theta = N\sigma_r - 2Y/(\theta_s-\alpha)$, $N = \frac{\theta_s+\alpha}{\theta_s-\alpha}$, $Y_0 = \frac{2Y}{\theta_s-\alpha}$ and $\frac{\partial v}{\partial r}+\frac{v}{r}=\Lambda\theta\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)$.

The latter is dilatancy relation for plastic rates given in the terms of solid velocities. Besides α, Λ are friction and dilatancy coefficients, Y is cohesion, $\theta_s = \mathrm{sgn}(\sigma_r - \sigma_\theta)$, $\theta = \mathrm{sgn}(\partial v/\partial r - v/r) = \theta_s$, the matrix tension is positive.

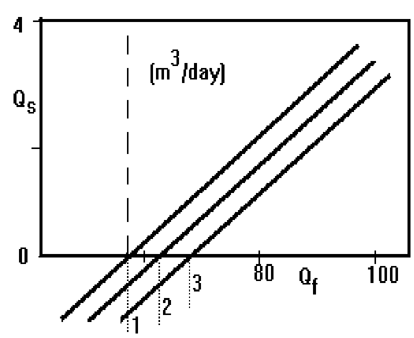

Fig.1. Sand production beginnings Q_f^* (1, 2, 3) depend on cohesion Y_o

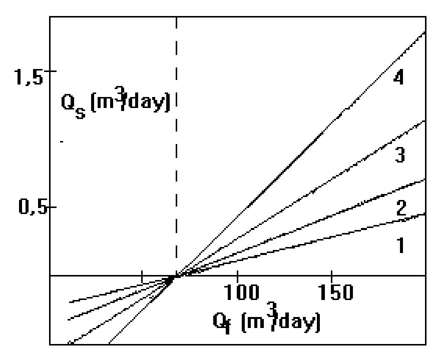

Fig.2. Sand production coefficient (1 – 4) depends on ν –dilatancy rate

The mass balances for the fluid and the matrix (ρ_f, ρ_s - the density of fluid and the intact solid material) are given as

$$\frac{\partial}{\partial t}(\phi\rho_f) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho_f\phi w) = 0, \qquad \frac{\partial}{\partial t}(1-\phi)\rho_s + \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho_s(1-\phi)v\right)=0 \quad (2)$$

The simplest stationary problem of "sand production" via well can be solved by assumption of coincidence of the plastic boundary with "feeding contour" R. Then

$$v=\frac{C}{r^n},\ n=\frac{1+\theta\Lambda}{1-\theta\Lambda},\quad v(r)=-\frac{Q_S}{2\pi hR(1-\varphi)},\ \frac{1-\varphi_R}{1-\varphi}=\left(\frac{R}{r}\right)^n,\qquad w(r)=-\frac{Q_f}{2\pi hr\varphi}$$

The connection between sand and fluid rates is calculated and given in Figures 1 and 2.

To avoid the concept of feeding contour is possible if one considered the self-similar problem of the opening of a well drilled to the infinite seam under initial pressure. The solution will depend just on the Boltzman variable:

$$\xi=\frac{r}{\sqrt{t}},\qquad \frac{\partial}{\partial t}=-\frac{1}{2t}\xi\frac{d}{d\xi},\qquad \frac{\partial}{\partial r}=\frac{1}{\sqrt{t}}\frac{d}{d\xi}$$

It is described by the system of ordinary equations but a bit more complicated than in the pore pressure diffusion case.

The ecology rules need in wastes disposal in the underground space. Here the injection of slurry through a well into the artesian seam is considered as plastic flow into a weak layer under high pore pressure. The axial symmetry of the problem assumes frontal stable displacement of masses. The selection of parameters values corresponds to the field data at Alaska experience.

The displacement instability is the main problem of the injection process as happens in the fracpack operation used for injection of gravel into an aquifer potentially dangerous for sand production (a variant of hydrofracturing of brittle rocks, which increases drastically a well productivity). The example of the first adequate calculation done numerically by the same model is shown in Figure 3. Here the stress concentration zones and thickening of a fracture, being filled with gravel (through the cut in a well tube), can be seen.

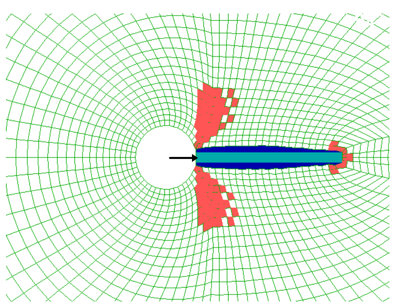

Figure 3. Initial fracture laterally growth (blue) and damage (red) zones under Frac & Pack

Nikolaevskii V.N. (1971) “Governing equations of plastic deformation of a granular medium”. J. Appl. Math. Mech. (PMM), # 6, 1017 – 1029.

Nikolaevskij V.N. (1990). “Mechanics of Porous and Fractured Media.” Singapore: World Scientific Publishers, 472 pp.

Nikolaevskiy V.N. (1996). “Geomechanics and Fluidodynamics. With Applications to Reservoir Engineering”. Dordrecht: Kluwer Acadmic Press, 348 pp.