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Elastic Interaction of Multiple Delaminations in Laminated Structures

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Summary The elastic interaction of multiple delaminations in laminated structures subject to out of plane loading has been investigated. This has been done by utilizing beam theory approximations of elasticity. Shielding and amplification of the energy release rate of the cracks has been quantified for the case of a cantilever beam. Results show important short and long range interactions between these cracks, depending mostly on their transverse spacing. The results have some similarity to those found by other investigators for the interaction of cracks in infinite bodies, but with strong modification of certain characteristics by mode ratio and thickness effects. The shielding and amplification effects strongly influence the propagation of the system of cracks leading to local instabilities, local strain hardening and crack arrest. The results are being validated using finite element solutions.

INTRODUCTION

Laminated structures are found in many man-made systems, such as aeronautical, aerospace and civil structures. They are also found in natural systems, such as seashells and insect cuticles. During manufacture (or growth) or under service conditions, these systems may develop delaminations that have the potential to reduce the stiffness and structural capacity of the system or to grow and cause premature and often catastrophic failures. Depending on the internal structure and loading conditions, the delaminations may tend to localize into a single crack or to spread into a diffuse region of multiple cracks leading to different macrostructural responses. The problem of quasi-static and dynamic delamination of structures in the presence of one single crack has been studied extensively, accounting also for cohesive and bridging mechanisms (see for instance [3,4] for some work of the authors). This paper deals with the problem of multiple delamination fracture.

An important phenomenon in delamination failure of laminated structures is the interaction between the cracks. An example is the interaction of a single crack with a field of micro-cracks, which can simulate damage in materials such as concrete or rock (see for instance [1-2]). This type of problem has introduced the concept of crack tip shielding and amplification, referring to the tendency of the cracks to either decrease or increase the stress intensification at the tips of the other cracks in the system. The problem of the interaction of multiple delaminations in structures subject to static, dynamic and impact loading has not yet been solved. In these structures, shielding and amplification phenomena are strongly controlled by structural effects that may alter the results previously obtained for other material systems.

DISCUSSION

The problem under consideration is a beam or plate, with n multiple through-width cracks of arbitrary length and transverse spacing. The material is assumed to be homogeneous, isotropic, and linear elastic. Figure 1 shows an example cantilever beam with multiple edge cracks, subjected to a static, concentrated force P at its end. The system is divided up into beam segments and Euler-Bernoulli beam theory is applied to determine the solution.

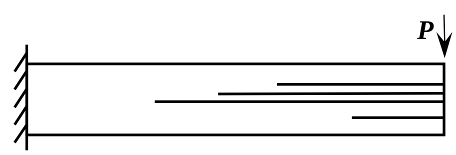

Figure 1 – Cantilever beam with multiple delaminations

The simplest configuration is when all of the cracks in the system are of the same length a and are assumed to propagate simultaneously. In this configuration the energy release rate for the extension of one of the cracks, $\mathcal{G}_i$, is easily determined from the total potential energy of the system. Equations (1) show $\mathcal{G}_i$ corresponding to n equally spaced cracks (1.a) and to n arbitrarily spaced cracks (1.b):

$$\text{(a)}\quad \mathcal{G}_i = \frac{6P^2a^2}{Eh^3}(n+2) \qquad \text{(b)}\quad \mathcal{G}_i = \frac{1}{2n}\frac{P^2a^2}{EI_0}\left(I_0\Big/\sum_{n+1} I_k - 1\right) \tag{1}$$

where E is the Young's modulus, h is the height of the beam, I_k (k=1,..n+1) is the moment of inertia of the beams in the cracked region and I_0 is the moment of inertia of the intact beam. Results show that as the number of equally spaced, equal length cracks increases so does $\mathcal{G}_i$ (diffuse damage), whereas, if the cracks are concentrated into a small layer (localized damage) then as the number of cracks increases, $\mathcal{G}_i$ decreases.

In order to analyze general crack configurations, the assumptions of equal length cracks and simultaneous propagation have been relaxed and the system of two cracks shown in Fig. 2 has been considered. Two solutions have been derived, the first valid when the upper crack is longer than the lower crack, $a_U > a_L$ and the second valid when $a_L > a_U$.

When the upper crack is longer, $a_U > a_L$, contact arises between beam segments 3 and 4. This contact has been

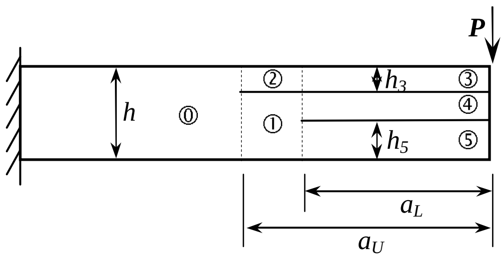

Figure 2 – Beam with two cracks

accounted for in three ways. The first, *A*, is to allow the beams to interpenetrate. The second, *B*, is to assume that the deflections of the beams in the cracked region are the same, thus preventing interpenetration (constrained model). The third, *C*, is to approximate contact with distributed linear springs that resist interpenetration. The stiffness of the springs depends on the transverse elasticity and heights of the two beam segments. This is considered the *exact* solution in the realm of beam theory. Contact between beams 3 and 4 also causes contact between beams 4 and 5, which is approximated similarly. The sizes of the contact regions are unknown *a priori*; therefore the problem is non-linear. When the lower crack is longer, $a_L > a_U$, there is opening between beam segments 4 and 5. In this case no additional assumptions are required.

The energy release rates for the upper, $\mathcal{G}_U$, and the lower, $\mathcal{G}_L$, cracks are calculated using the J-integral around paths surrounding each crack tip. Figure 3.a shows $\mathcal{G}_L$ normalized by the energy release rate for the same crack in the absence of the upper crack, $\mathcal{G}_{Lo}$, for one set of parameters. When the upper crack is longer (left side of the diagram), solutions *A* and *B* are upper and lower bounds of the exact solution *C*. When the upper crack is shorter (right side), all solutions coincide. For this set of parameters, $\mathcal{G}_L$ is always amplified. This amplification is strong when the lower crack is shorter. Also visible is a jump in $\mathcal{G}_L$ when the cracks are of equal length. This jump is similar to what was observed in [1] for the interaction of a semi-infinite main crack with a pair of micro-cracks in an infinite medium.

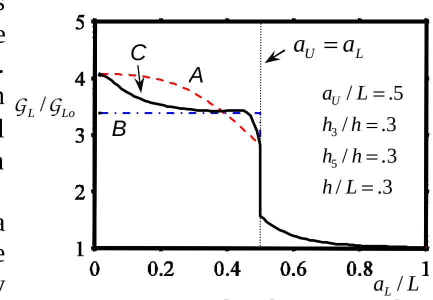

Figure 3a - Normalized Energy Release Rate of Lower Crack

Figure 3b shows contours of regions of amplification and shielding as a function of the transverse positions of the two cracks. If the positions fall in the lower left regions, e.g., point (a), then $\mathcal{G}_L$ will always be amplified (Fig. 3a). If they fall in the shaded region, point (b), then there will be a mixture of amplification and shielding depending on the relative length of the two cracks. Finally, if the positions fall in the upper region, point (c), then $\mathcal{G}_L$ will always be shielded.

When the cracks have similar lengths, beam segment 1 becomes stocky and the validity of Euler-Bernoulli beam theory is questionable. The importance of shear deformation has been investigating by using Timoshenko beam theory. The results show that for sufficiently long cracks the shear deformations of beam segment 1 have no effect on the macrostructural response and $\mathcal{G}_i$.

A study of the quasi-static propagation of the two-crack system has yielded contour maps that highlight different crack propagation mechanisms controlled by the crack configuration. It has also revealed many interesting effects. For instance, for certain crack geometries, there is a local strain hardening behavior due to the shielding effect. For other geometries, there is crack pull-along, in which one crack will begin to propagate, and then some distance later the second crack will begin to propagate as well. The jumps in the energy release rate (Fig. 3) can also be seen in the quasi-static load deflection curve as local snap-back instabilities.

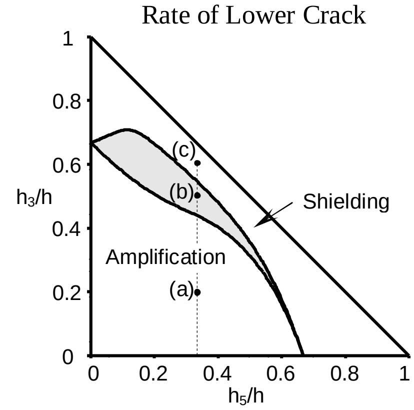

Figure 3b –Regions of Shielding and Amplification

CONCLUSIONS

The study has shown that there are significant short and long range interaction effects between cracks in a structure with multiple delaminations. These interaction effects can be shielding or amplification of the energy release rate of a crack and strongly depend on the geometry of the system. The results are currently being validated using finite element solutions. Additional effects, such as friction in the regions of crack face contact and the effect of cohesive and bridging mechanisms acting along the cracks are under investigation.

Acknowledgments

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