Nonlinear Dynamic Stability of Multi-Suspended Roof Systems

by
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Introduction

Suspended roof systems are extensively used in many structural applications such as commercial halls, sport centers, airport halls, trade and exhibition centers, etc. Inspired engineers as L. Mies van der Rohe, K. Tange, P. L. Nervi, and others have designed and build numerous great buildings with suspended roofs as their main structural component. The last 10 to 15 years, the development of powerful computers and sophisticated nonlinear FEM software (ADINA, ABAQUS, etc.) has enabled engineers to utilise suspension roofs in complicated large scale structures, some of which can be classified among unique examples of engineering excellence.

A large number of recent studies are concerned with the investigation of the dynamic buckling and stability of simple 2-DOF and 3-DOF initially imperfect models with damping under step loading, that simulate single and double suspension roofs and important findings have been reported. The present work deals with the fully nonlinear static and dynamic stability analysis of a simplified multi-DOF initially imperfect dissipative model, upon which a constant directional (conservative) vertical step loading of infinite duration is applied.

Mathematical formulation

Let us consider the dissipative, initially imperfect multi-DOF system shown in Fig. 1.

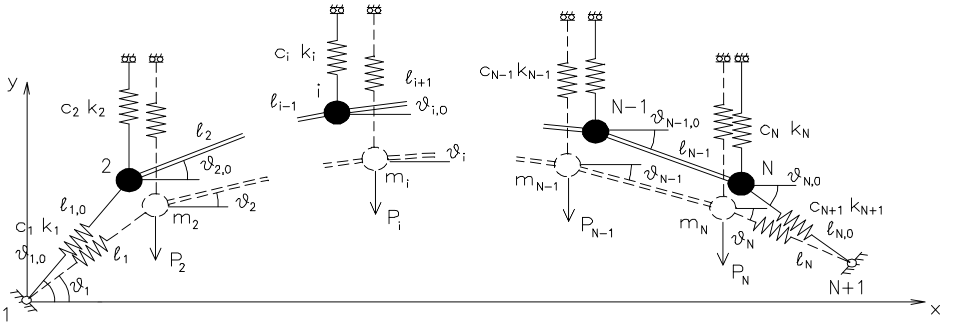

Fig. 1. A N-DOF *autonomous dissipative system under step loading, simulating a multiple suspension roof.*

The system is initially at rest in a configuration described as follows

$$x_{i,0} = \ell_{1,0}\cos\theta_{1,0} + \sum_{j=2}^{i} \ell_j \cos\theta_{j,0} \qquad\qquad y_{i,0} = \ell_{1,0}\sin\theta_{1,0} + \sum_{j=1}^{i} \ell_j \sin\theta_{j,0} \tag{1}$$

where $x_{i,0}$ and $y_{i,0}$ are the initial coordinates of joint i, and ℓ_j and $\theta_{j,0}$ are the length and the initial direction of bar j, respectively. After applying the external forces P_i at the joints (i=1,...,N), the deformed configuration of the system is described as follows

$$x_i = \ell_1\cos\theta_1 + \sum_{j=2}^{i} \ell_j \cos\theta_j \qquad\qquad y_i = \ell_1\sin\theta_1 + \sum_{j=2}^{i} \ell_j \sin\theta_j \tag{2}$$

where x_i and y_i are the coordinates of joint i, and θ_j is the direction of bar j in the deformed position. The strain energy U of the system is

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$$U = \tfrac{1}{2}k_1(\ell_1 - \ell_{1,0})^2 + \tfrac{1}{2}\sum_{i=2}^{N} k_i(\ell_1 \sin\theta_1 + \sum_{j=2}^{i-1} \ell_j \sin\theta_j - \ell_{1,0}\sin\theta_{1,0} - \sum_{j=2}^{i-1}\ell_j \sin\theta_{j,0})^2 +$$
$$+\tfrac{1}{2}k_{N+1}\{[(\ell_{1,0}\cos\theta_{1,0} + \sum_{j=2}^{N-1}\ell_j\cos\theta_{j,0} + \ell_{N,0}\cos\theta_{N,0} - \sum_{j=1}^{N-1}\ell_j\cos\theta_j)^2 + \quad (3)$$
$$+(\ell_{1,0}\sin\theta_{1,0} + \sum_{j=2}^{N-1}\ell_j\sin\theta_{j,0} + \ell_{N,0}\sin\theta_{N,0} - \sum_{j=1}^{N-1}\ell_j\sin\theta_j)^2]^{\frac{1}{2}} - \ell_{N,0}\}$$

The work Ω done by the external forces is

$$\Omega = -\sum_{i=2}^{N} P_i(\ell_1\sin\theta_1 + \sum_{j=2}^{i-1}\ell_j\sin\theta_j - \ell_{1,0}\sin\theta_{1,0} - \sum_{j=2}^{i-1}\ell_j\sin\theta_{j,0}) \quad (4)$$

The kinetic energy K of the system is

$$K = \tfrac{1}{2}\sum_{i=2}^{N} m_i[(\dot\ell_1\cos\theta_1 - \ell_1\dot\theta_1\sin\theta_1 - \sum_{j=2}^{i-1}\ell_j\dot\theta_j\sin\theta_j)^2 + (\dot\ell_1\sin\theta_1 + \ell_1\dot\theta_1\cos\theta_1 + \sum_{j=2}^{i-1}\ell_j\dot\theta_j\cos\theta_j)^2] \quad (5)$$

and the dissipation energy F is

$$F = \tfrac{1}{2}c_1(\dot\ell_1^2 + \ell_1^2\dot\theta_1^2) + \tfrac{1}{2}\sum_{i=2}^{N} c_i(\dot\ell_1\sin\theta_1 + \sum_{j=1}^{i-1}\ell_j\dot\theta_j\cos\theta_j)^2 + \tfrac{1}{2}c_{N+1}[(\dot\ell_1\cos\theta_1 - \sum_{j=1}^{N-1}\ell_j\dot\theta_j\sin\theta_j)^2 + (\dot\ell_1\sin\theta_1 + \sum_{j=1}^{N-1}\ell_j\dot\theta_j\cos\theta_j)^2] \quad (6)$$

The equations of motion of the system are derived using the Lagrange equation

$$\frac{\partial}{\partial t}\left(\frac{\partial K}{\partial \dot\ell_1}\right) - \frac{\partial K}{\partial \ell_1} + \frac{\partial F}{\partial \dot\ell_1} + \frac{\partial V}{\partial \ell_1} = 0, \quad \frac{\partial}{\partial t}\left(\frac{\partial K}{\partial \dot\theta_i}\right) - \frac{\partial K}{\partial \theta_i} + \frac{\partial F}{\partial \dot\theta_i} + \frac{\partial V}{\partial \theta_i} = 0, \quad i = 1,2,\ldots,(N-1) \quad (7)$$

Numerical results and discussion

In the numerical examples that follow, a 4-DOF system is employed (i.e. N=4) with the following characteristics: $\theta_{1,0}=\theta_{4,0}=30^o$, $\theta_{2,0}=-\theta_{3,0}=10^o$, $k_1=k_i=k_5=1$, $c_1=c_i=c_5=0.1$, $\theta_{2,0}^*=-5^o$, $\theta_{3,0}^*=7^o$, P=0.5, producing the following plots:

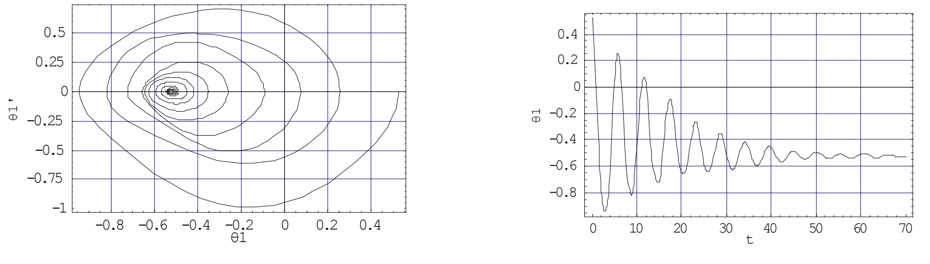

Fig.2 Phase-plane portrait $\dot\theta_1 - \theta_1$, and (d) time series θ_1 vs. t

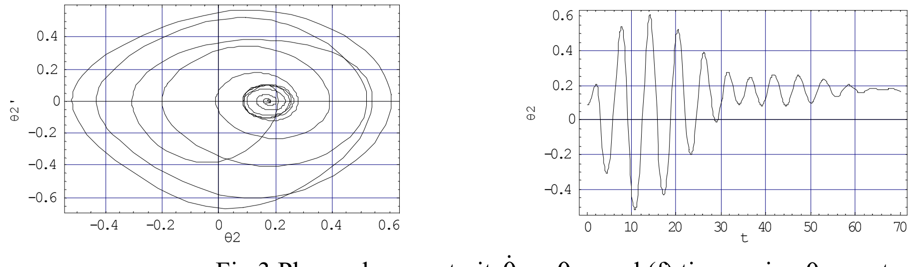

Fig.3 Phase-plane portrait $\dot\theta_2 - \theta_2$, and (f) time series θ_2 vs. t

From these plots it is directly concluded that for all model cases dealt with the global dynamic response is stable, which is the primary advantage of multi-suspension roofs, successfully captured by the proposed simulation.

References

[1] Blaser W, Stucky M. Drawings of great buildings. Boston: Birkhauser Verlag, 1983.

[2] Michaltsos G.T., and Sophianopoulos D. S., Nonlinear stability of a simplified model for the simulation of double suspension roofs, *Engineering Structures*, **23**, 705-14, 2001.