

## COUPLING OF AXISYMMETRIC AND 3D SHELLS MODELS FOR NON LINEAR ELASTOPLASTIC BUCKLING PREDICTION OF MAINLY AXISYMMETRIC SHELLS

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**Summary** This paper describes a method which enables to couple 3D shell simulation with axisymmetric models in order to predict the non linear buckling for shells which have localised non axisymmetric geometries, or loadings.

The axisymmetric models are very efficient and powerful for prediction of non linear buckling of axisymmetric shells with non localised imperfections or loadings. The idea is simply to develop the response on a Fourier basis and to use a finite number of Fourier modes to approximate the solution. This strategy has been extensively developed and successfully used for the analysis of axisymmetric perfect or imperfect shells [1], [2], [3] subjected to any load which is slowly spatially varying. When there is a local geometrical imperfection (say a hole) or a local quickly varying load or local support condition the axisymmetric models are not really efficient. The usual way to study the problem is to use a 3D shell model. This method leads to very heavy meshes if one wants to get precise predictions. The idea developed in the paper is to propose a method to combine a 3D analysis for the region where a local strong discontinuity is present and an axisymmetric coupled Fourier analysis for the rest of the structure. The idea was first introduced by Gould [4] and some results were shown for linear elastic response. We have basically to couple two subdomains each having its own discretisation. We shall use the "mortar" method [5] to couple the two subdomains. Subdomain 1 has an axisymmetric representation whereas sub domain 2 has a 3D shell representation. We have used and augmented Lagrangian formulation to express the energy of the coupled problem:

$$W^{tot} = W^{3D} + W^{axis} + W^{interface} \quad (1)$$

In equation (1) the energy is split in a 3D part an axisymmetric part and an interface one. On the interface we have two different displacement approximations and a third Lagrange multiplier approximation. As in [5] the interface energy can be expressed by:

$$W^{interface} = \int_{interface} \underline{\Lambda}^t (\underline{U}^{axi} - \underline{U}^{3D}) ds \quad (2)$$

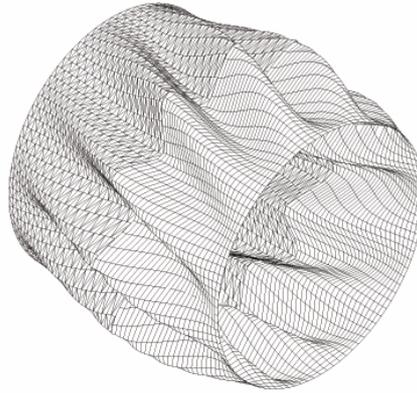
in this equation  $\underline{\Lambda}$  is the Lagrange multiplier vector,  $\underline{U}$  are the axisymmetric or 3D displacement vectors at the interface. As usual in mortar method the discretisation of Lagrange multiplier (number of Lagrange multipliers and shape functions) are chosen on either with the 3D support or with the axisymmetric one. The Babuska Brezzi condition is used to choose the best representation of Lagrange multiplier on the interface. It is shown that the 3D choice leads to locking of the interface, and that it is best to choose axisymmetric representation of Lagrange multipliers. The method can also be applied introducing interface compliance. Equation (2) can then be replaced by:

$$W^{interface} = \frac{1}{2} \int_{interface} w^{def} (\underline{U}^{axi} - \underline{U}^{3D}) ds \quad (3)$$

in equation (3)  $w^{def}$  is the strain energy density of the interface. The interface is supposed to behave elastically and is a small ring having the current thickness  $t$  of the shell and a very small height  $h$ . Here again the interface energy can be discretised either with a 3D model or with an axisymmetric one. As in the previous one it is shown that 3D representation leads to locking of the interface because of an over-constrained problem.

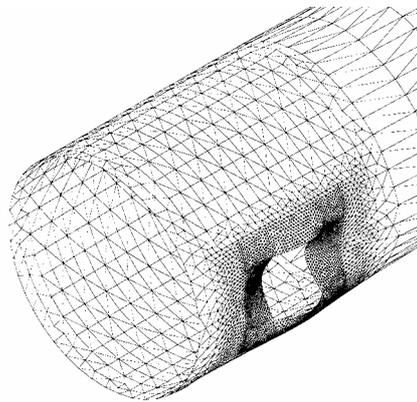
The method has been applied to basic examples as the torsional buckling of a cylinder one half (the left part on figure 1) of it being meshed with 3D DKT Shell elements and the other part being meshed in axisymmetric. (only two Fourier modes ( $n$  symmetric and  $n$  anti symmetric) are sufficient to represent the buckling mode). The computed torsion load is 0.5% higher than the theoretical one.

The computed mode is displayed on figure 1



**Figure 1:** torsion buckling mode

The second example is devoted to the prediction of non linear buckling of a chimney with a square hole at its basis under bending load. The non linear buckling mode is displayed on figure 2, and it compares well with a full 3D analysis.



**Figure 2:** chimney with hole under bending with 3D-2D coupled analysis

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