OVERALL PROPERTIES OF PERIODIC BIOCOMPOSITES

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Summary Electroelastic properties of two-phase circular cylindrical fibre-reinforced biocomposites are calculated for square and hexagonal periodical arrays using the asymptotic homogenization method. The materials of the composite belong to the hexagonal system, classes 622 and 6. Easy to compute closed-form formulae are obtained. Numerical results are shown for several biomaterials, e.g., collagen and collagen-hydroxyapatite composite and others.

INTRODUCTION

Composites, whether natural or man-made, span a wide variety of geometries (particulates, fibers, laminates, etc.). Composition is guided, and even tailored, by the application at hand. In medical applications the constituents must fulfill a number of prerequisites to be of any use [1]. At present there are a number of biomaterials, as they are called, under study for, say, bone regeneration, implants, replacement scaffolds, etc. [2]. Electroelastic properties of two of them, collagen and collagen-hydroxyapatite, have been measured lately [3]. Their anisotropy belongs to the hexagonal system, class 622. Other related biomaterials (bone, tendon, etc.) have the symmetry 6. It is of interest to find the effective properties of composites with these elements. Recently, the overall properties of two-phase fiber-reinforced periodic composites, i.e., long parallel circular cylindrical fibers of one material embedded into another one, have been found in closed-form for square [4] and hexagonal [5] arrays and materials with the 6mm symmetry. These were found using the asymptotic homogenization method. Here it is proposed to use that method as to include the classes 622 and 6. For the purpose of brevity, only one such problem (hexagonal array and 622 symmetry) is explicitly addressed here.

STATEMENT OF THE PROBLEM

Figure 1 shows the unit cell. As a consequence of the geometrical axial symmetry, the initial set of equations uncouple into two independent systems. For brevity, the simpler problem is described here, since it typifies the method and some of the results. This corresponds to the case of out-of-plane mechanical displacement/in-plane electric field. The governing equations are

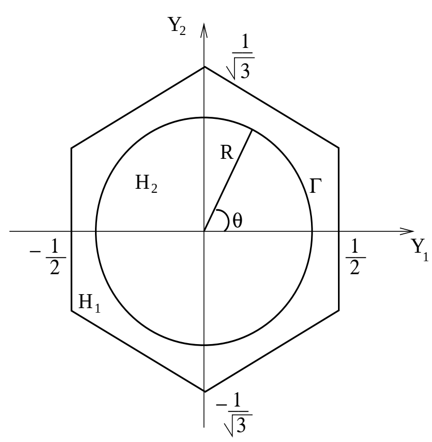

Figure 1: Hexagonal Unit Cell

$$\text{div}\,(\sigma_{13}^{(\epsilon)}, \sigma_{23}^{(\epsilon)}) = 0 = \text{div}\,(D_1^{(\epsilon)}, D_2^{(\epsilon)}) \text{ in } R^2, \tag{1}$$

where $\sigma_{13}^{(\epsilon)}, \sigma_{23}^{(\epsilon)}$ are the stress components; $D_1^{(\epsilon)}, D_2^{(\epsilon)}$ those of the electric displacement and $\epsilon \ll 1$. The constitutive relations for the class 622 are

$$\sigma_{23} = 2p\epsilon_{23} - s'E_1,\ \sigma_{13} = 2p\epsilon_{13} + s'E_2, \tag{2}$$

$$D_1 = 2s'\epsilon_{23} + tE_1,\ D_2 = -2s'\epsilon_{13} + tE_2, \tag{3}$$

where p, s' and t are the composite's axial shear modulus, stress piezoelectric constant and axial permittivity, respectively. The same notation is used for the properties of the phases except that an index 1 (2) is used for the matrix (fiber). The components of the strain are $\epsilon_{13}, \epsilon_{23}$ and those of the electric field E_1, E_2.

Asymptotic homogenization method

The method proceeds as usual [6]. The focus here is on the effective properties p, s' and t. They are given by

$$p = p_v + \langle pM_{,1} - s'N_{,2}\rangle,\ s' = s'_v + \langle s'M_{,1} + tN_{,2}\rangle, \tag{4}$$

where the comma notation denotes differentiation with respect to y_1 or y_2, the index v indicates the Voigt (arithmetic) mean, e.g., $p_v = (1 - V_2)p_1 + V_2 p_2$, where V_2 is the area fraction occupied by the fiber and $|H|\langle F(y)\rangle = \int_H F(\mathbf{y})da, H = H_1 \cup H_2$ is the unit cell (Fig. 1). As for t, it is obtained from the universal relation derived using the Milgrom-Shtrikman compatibility equation, since it connects p, s' and t. The functions $M^{(\gamma)}$ and $N^{(\gamma)}, \gamma = 1, 2$, are the solution of the following unit cell problem, the so-called local problem:

$$\Delta M^{(\gamma)} = \Delta N^{(\gamma)} = 0 \text{ in } H = H_1 \cup H_2, \; \| M^{(\gamma)} \| = \| N^{(\gamma)} \| = 0 \text{ on } \Gamma, \tag{5}$$

$$\| (p_\gamma M^{(\gamma)}_{,2} - t_\gamma N^{(\gamma)}_{,1})n_1 + (p_\gamma M^{(\gamma)}_{,2} + s'_\gamma N^{(\gamma)}_{,1})n_2 \| = - \| p_\gamma \| n_1, \text{ on } \Gamma, \tag{6}$$

$$\| (s'_\gamma M^{(\gamma)}_{,2} - t_\gamma N^{(\gamma)}_{,1})n_1 - (s'_\gamma M^{(\gamma)}_{,1} + t_\gamma N^{(\gamma)}_{,2})n_2 \| = \| s'_\gamma \| n_2, \text{ on } \Gamma, \tag{7}$$

$$\langle M \rangle = \langle N \rangle = 0, \tag{8}$$

where the contrast to p across Γ is $\| p_\gamma \| = p_1 - p_2$, etc., (n_1, n_2) is the unit normal to Γ. Potential methods are used to find $M^{(\gamma)}$ and $N^{(\gamma)}$ using doubly periodic harmonic functions of periods $1, \exp(i\pi/3)$ (the quasi-periodic Weierstrass zeta function and its derivatives of order k). The application of Green's theorem to (4), yields

$$p = p_1(1 - 2\pi a_1), s' = s'_1 + 2\pi t_1 b_1, \tag{9}$$

where a_1, b_1, are the residues at the origin of the functions $M^{(1)}$ and $N^{(1)}$, respectively. They can be found after solving an infinite system of algebraic equations that converge very quickly. Its truncation to a sequence of finite systems leads to the sought coefficients a_1, b_1 and the overall properties (4). The numerical scheme ends when enough accuracy is achieved.

NUMERICAL EXAMPLES

The material properties for the calculations that follow were taken from [3]. After some minor calculations, the data used is: for collagen, taken as the matrix medium $p_1 = 1.4 \; GPa, t_1/\varepsilon_0 = 2.825(\varepsilon_0 = 8.854 \times 10^{-12} C^2/Nm^2$, permittivity of free space), $d_1 = 6.2.10^{-14} C/N$; the fiber material is a collagen-hydroxyapatite (HA) composite, whose properties are $p_2 = 2.697 \; GPa, t_2/\varepsilon_0 = 2.509, d_2 = 4.1. \times 10^{-14} C/N$. The results are shown in Fig. 2, which displays a plot of the overall properties p, t/ϵ_0 and d as a function of the fiber volume fraction V_2, up to the percolation limit (i.e., when the fibers get in contact). The three properties show a monotonic behaviour as a function of the fiber volume fraction. Note that $d = s'/p$ is the strain piezoelectric coefficient.

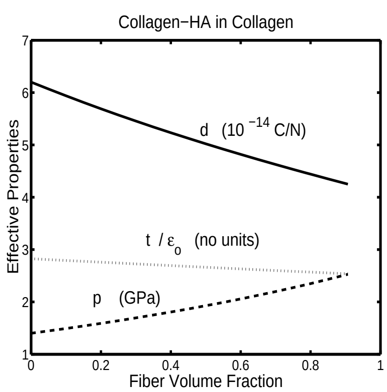

Figure 2: Overall Properties

CONCLUSIONS

Closed-form formulae for the overall properties of fiber-reinforced composites, whose electroelastic characteristics belong to the hexagonal system, classes 622 and 6, are obtained for square and hexagonal arrays. An example using newly measured properties of collagen and a collagen-hydroxyapatite composite is shown here. Other examples will be shown.

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