THE SPACING OF LANGMUIR CIRCULATION IN STRONG WAVY SHEAR

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The inviscid instability of $O(\epsilon)$ two-dimensional free surface gravity waves propagating along an $O(1)$ parallel shear flow is considered. The modes of instability involve spanwise-periodic longitudinal vortices resembling oceanic Langmuir circulation. Here, not only are wave-induced mean effects important but also wave modulation, caused by developing mean streamwise-velocity anomalies. The former is described by the generalized Lagrangian-mean formulation of Andrews & McIntyre and the latter by a modified Rayleigh equation given by Craik. Since both effects are essential, the instability is called "generalized" Craik-Leibovich (CLg) to distinguish it from the related but fundamentally different CL2 instability. Motivation for the work stems from interest in the air-sea interface and in particular in modeling Langmuir circulations. These occur in open water bodies in the presence of moderate winds and play an important role in the transfer of heat, mass and momentum between the atmosphere and the ocean. CL2 is a leading candidate mechanism to describe this occurrence but there is as yet no proof that CL2 is realizable in the ocean or laboratory.

According to the Craik-Phillips-Shen criterion, wave mean interactions are unstable to longitudinal vortex form if, from the reference frame of the waves, and in the direction of increasing mean flow, the relative increase in mean flow exceeds the relative increase in wave amplitude. So, the flow under consideration is unstable to longitudinal vortex form provided the amplitude of the waves diminishes with depth. Of specific interest is whether spanwise distortion of the wave field acts to enhance or inhibit instability to longitudinal vortices. Also of interest is whether the instability gives rise to a preferred spacing for the vortices and whether that spacing concurs well or poorly with experiment.

The layer depth is varied from much less than the e-folding depth of the $O(\epsilon)$ wave motion, to infinity. Relative to an identical shear flow with rigid though wavy top boundary, it is found that wave modulation acts to increase the maximum growth rate of the instability. It is further found that, while mean streamwise-velocity anomalies in layers of finite depth are typically a maximum at the free surface and of single sign with depth, anomalies of mixed sign are common when the depth is infinite. Mixed sign is associated with multiple, vertically stacked vortices, a feature expected to enhance mixing. Finally, the preferred spacings calculated herein concur well with those observed in laboratory experiments, with the implication that the instability acting in the experiments very likely is CLg.

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