XXI ICTAM, 15-21 August 2004, Warsaw, Poland

DERIVATION OF THE HIGHER-ORDER STIFFNESS MATRIX OF A SPACE FRAME ELEMENT FOR GEOMETRIC NONLINEAR ANALYSIS OF STRUCTRUES

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Summary A physical concept, the rigid body rule, is applied for the derivation of the higher-order stiffness matrix of a space frame element. The derivation has a physical meaning that is the higher-order stiffness matrix can be derived by regarding there is a set of incremental nodal forces existing on the element, then the element undergoes a small rigid body rotation. The incremental forces should keep their magnitude and follow the rigid body motions. Then taking advantage of the existing geometric stiffness matrix derived by researchers, the higher- order stiffness matrix can be analogy derived without any difficulty. The derived higher-order stiffness matrix has explicit expressions. It can be used at the forces recovery stage in the geometric nonlinear analysis of frame structures. Meanwhile an effective numerical method, the Generalized Displacement Control (GDC) method, was adopted to trace the load-defection curves of the structures. Some numerical examples were tested by taking the proposed higher-order stiffness matrix into consideration in the nonlinear analysis of the structures **.**

INTRODUCTION

Researchers have paid a lot of attention during the past four decades to conduct the geometrical nonlinear analysis of framed structures (Bathe [1]; Yang and Kuo [2]; Belytschko et al. [3]; Yang, Kuo and Wu, [4]) and others. Most of them have derived the geometric stiffness matrix for the geometrical nonlinear analysis. In this study, the rigid-body-motion concept was used in the derivation of the higher-order stiffness matrix of a frame element. One can use the derived higher-order stiffness matrix combining linear stiffness and geometric stiffness matrices of the element in the force recovery stage to dramatically reduce the iterations in each incremental step during the whole geometrical nonlinear analysis.

In the literature, based on the various variational principles, many geometric nonlinear elements have been generated by researchers, which included truss, frame and plate elements, and so on. When talking about geometric nonlinear analyses, without sayings, linear and geometric stiffness matrices are presented in the formulations no matter what kinds of variational principles were adopted by researchers. In the meanwhile, based on incremental variatonal principles, different forms of geometric stiffness matrices have been generated. But for the higher-order stiffness matrix less attention has been made because of its tedious derivations and lengthy expressions by conventional derivations. In this research an explicit form for the higher-order stiffness matrix will be derived by extension the concept of the so call the rigid body test.

In the nonlinear finite element method for a formulated nonlinear element, a criterion used to qualify the derivation geometric nonlinear elements, so called the rigid body test that was first proposed by Yang and Chiou [5] in 1987. It can be stated as followed: Once an element is in equilibrium with a set of initial forces acted when it undergoes rigid body motions, the forces acted on the element will keep their magnitudes and rotate with the rigid body motions. It is named as rigid body rule that is a physical interpretable concept. Using this concept, an analogy from of the higher-order stiffness can be easily established for nonlinear analysis of structures.

The derived higher-order stiffness matrix has a clear physical meaning that just as to the properties of the geometric stiffness matrix of the element. It can be stated as following. When there is a set of incremental nodal forces existing on the element, let the element undergo a small rigid body motion, the incremental forces should keep their magnitude and following the rigid body motions just as the initial nodal forces acting on the element behave. In this way the higher-order stiffness matrix of the frame element can be derived with the aids of the geometric stiffness matrix. One merely changes the initial forces in the geometric stiffness instead of the incremental forces. The higher-order stiffness matrix of the element can be formed easily. It has the same expression from but different in incremental forces in contrast to the initial forces in the geometric stiffness matrix. This formulation is simple and explainable. It can be seen that most coefficients lying in the higher-order stiffness matrix have explicit expressions just as the geometric stiffness matrix does.

The main purpose of this study is to derive the higher stiffness matrix of a space frame with the aids of widely derived geometric stiffness matrix and the concept of rigid body motions. The derived higher-order stiffness matrix combined the linear stiffness and geometric stiffness matrices to form a tangent stiffness matrix can be used in the force recovery stage in the geometrical nonlinear analysis. It can effectively reduce the number of iterations at each incremental step by use the derived higher-order stiffness matrix can be ensured. Finally some numerical examples were solved by the proposed elements and the generalized displacement control method proposed by Yang and Shieh [6].

NUMERICAL EXAMPLE

A deep circular arch structure was considered herein. The geometry and material properties of the deep circular arch are shown as in Figure 1. The circular arch is subject to a concentrated load on the convex side, as also depicted in Figure 1. One of the edges of the circular arch is hinged and the other is fixed. Herein 36 frame elements were also used to discrete the deep circular arch. One or two iterations were needed in each incremental step in the analysis. Results of the central deflection versus the applied load have been plotted in Figure 2, with convergent criterion by setting less than one thousandth of the relative norm of the unbalanced forces for per incremental step. The results obtained by the proposed higher-order stiffness matrix agree well with conventional method, i.e., only the linear stiffness matrix and geometric stiffness matrix are included in the tangential stiffness matrix of the element.

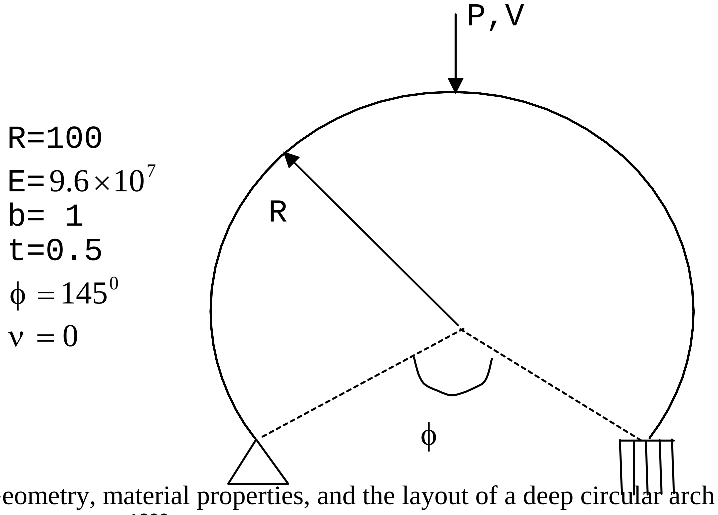

Figure 1. Geometry, material properties, and the layout of a deep circular arch under a concentrated load.

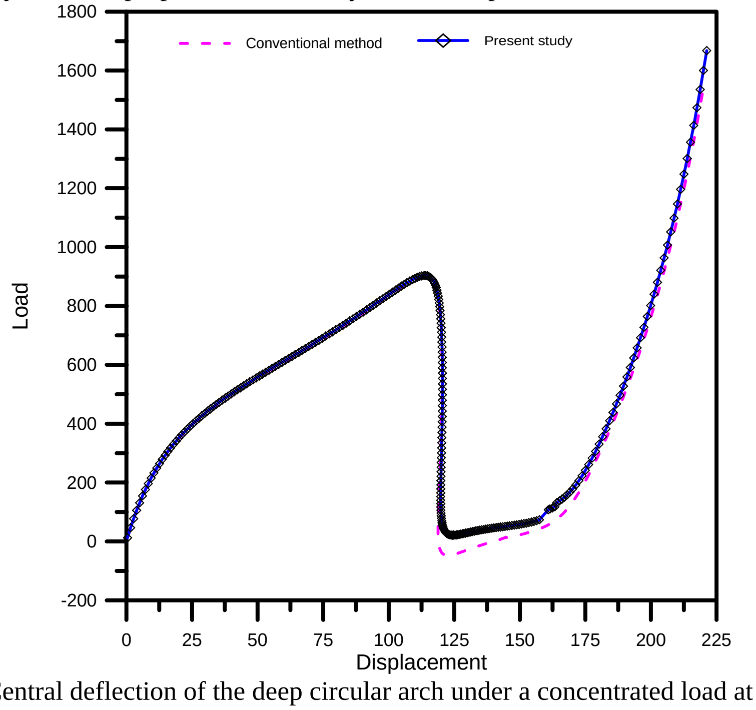

Figure 2. Central deflection of the deep circular arch under a concentrated load at its crown

CONCLUSIONS

From the demonstrated example and its results, it shows the reliable solution of the structure modelled by the proposed elements. This means that the present concept is applicable and has less iteration required in the force recovery stage for all the geometric nonlinear analysis of frame and arch structures. Meanwhile, the solution procedure is quite effective in dealing with limits points, snap through phenomena. The present approach has been referred as an engineer's approach, because the derivation is based on physically intuitive concepts. By the demonstrated example, the concept, i.e., the rigid body rule is power, not only can be adopted to derive a geometric stiffness matrix but also play a key concept for derivation of the almost an exactly tangential stiffness matrix of the frame element for the geometric non-linear analysis of frames and arches.

References

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