EXPERIMENTS ON ROTATING AND REFLECTING INTERNAL WAVE BEAMS

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Summary : We present new results for two sets of experiments on internal wave beams. For conical beams in a rotating system, we confirm the dependence of the beam angle on the rotation rate and identify the cut-off frequencies. For reflected beams, we provide the first experimental observations of second harmonic beams generated by nonlinear interactions at the wall. The synthetic schlieren method of visualisation was used for both experiments.

INTERNAL WAVE BEAMS

Internal waves arise as a result of disturbances in a stratified fluid. If gravity is assumed to be in the vertical direction, the dispersion relation is such that disturbances generated by a localised source propagate as beams. The angle of the beams with respect to the horizontal is

$$\theta = \sin^{-1}(\omega/N), \tag{1}$$

where ω is the excitation frequency and N is the Brunt-Vaisala frequency of the stratification. There is thus a cut off frequency ($\omega = N$) above which disturbances cannot propagate. This fascinating phenomenon was first investigated by Mowbray & Rarity [1] and has been the subject of a wide variety of research relevant to both the oceans and the atmosphere. Recently, the novel "synthetic schlieren" experimental technique [2] has enabled detailed quantitative investigations of the beam structure.

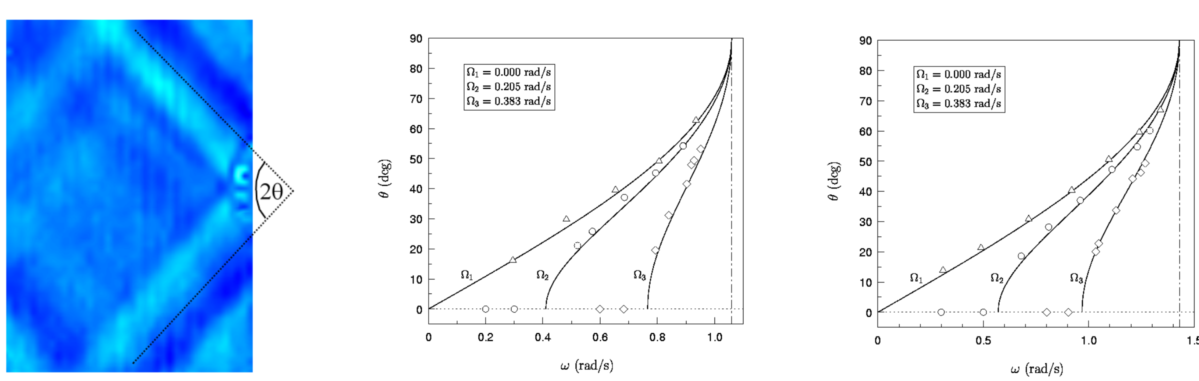
Rotating internal wave beams

The influence of rotation on the orientation of internal wave beams is documented by LeBlond & Mysak [3]. Disturbances from a point source generate a conical wave-beam structure whose angle to the horizontal is given by the dispersion relation

$$\theta = \sin^{-1}\sqrt{\frac{\omega^2 - 4\Omega^2}{N^2 - 4\Omega^2}}, \tag{2}$$

where Ω is the angular rotation rate of the system, that is oriented anti-parallel to gravity. In contrast to the non-rotating case, there are both lower (2Ω) and upper (N) cut-off frequencies, outside of which no disturbances can propagate. This dispersion relation has never previously been verified experimentally.

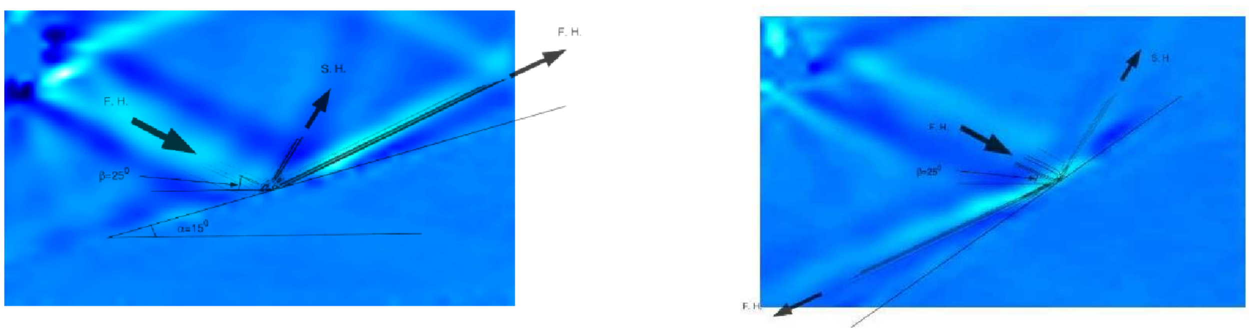
The experiment comprised an acrylic tank of diameter 1 meter mounted on a rotating table. Two stable salinity stratifications ($N = 1.06, 1.43 \pm 0.01\mathrm{s}^{-1}$) were established by filling the tank using the double bucket method. The density gradient was measured using a calibrated salinity probe. A sphere of radius 2.54±0.02 cm was fixed to a thin rod and positioned at the center of the tank. The sphere was oscillated vertically with an amplitude of 1.50±0.01cm over a range of frequencies. Through the arrangement of a video camera, a mirror and a back lit pattern of dots the resulting conical wave beams were visualised using the "synthetic schlieren" method [2]. A beam cross section for which $N = 1.43\mathrm{s}^{-1}$, $\Omega = 0.285 \pm 0.007\mathrm{s}^{-1}$ and $\omega = 1.11 \pm 0.01\mathrm{s}^{-1}$ is presented in Figure 1(a). The beam angles were objectively measured using the Radon transform of images such as that in Figure 1(a). Results for $N = 1.06$ and $N = 1.43$ are presented in Figures 1(b) and 1(c). The agreement between experiment and theory is very good, and for the rotating systems one can see the existence of a lower cut off frequency, below which no beams could propagate.



(a) (b) (c)

FIGURE 1 : (a) Cross section of a pair of conical wave beams originating from a sphere (center-right). $N = 1.43\mathrm{s}^{-1}$, $\Omega = 0.285\mathrm{s}^{-1}$, $\omega = 1.11\mathrm{s}^{-1}$ (b) Experimental results for $N = 1.06\mathrm{s}^{-1}$ (c) Experimental results for $N = 1.43\mathrm{s}^{-1}$.

Reflecting internal wave beams.
It has recently been realised that the linearised solutions for internal wave beams are, in fact, solutions of the full nonlinear problem [4]. The effect of nonlinear interactions between two such beams may be calculated using these linear solutions [5]. Such a situation arises when two distinct beams bisect within the body of a stratified fluid. Alternatively, interaction can arise when an individual beam is reflected from a boundary, in which case the incoming beam interacts with its reflected counterpart. These interactions have been predicted to generate additional beams, of different harmonics, whose angle of propagation is determined by the dispersion relation (1). If the interaction takes place at a sidewall, propagation of the reflected beams is also directed by the the angle of inclination of the wall ϕ.
The experiments were performed in a tank 30cm deep, 40cm high and 40cm wide. An acrylic cylinder 2.54±0.02cm in radius and 30cm long was attached to a slender rod and positioned across the center of the tank. A stable salinity stratification was obtained by filling the tank from below using a double bucket method. The cylinder was oscillated with an amplitude of 0.5 ± 0.01cm to generate planar internal wave beams emanating at an angle of 25.0 ± 0.1^o along the length of the cylinder. The arrangement was such that one of the beams was reflected from an inclined surface, and the reflections were observed using the synthetic schlieren set-up.
The visualisations for two experimental arrangements are shown in Figures 2(a) and 2(b). The beams originate from the cylinder in the top right hand corner of each image. Superimposed on top of the images are the results of complementary numerical simulations, which indicate the orientation of the incoming and reflected beams. The arrangement for Figure 2(a) was $\theta = 25.0 \pm 0.1^o$ and $\phi = 15.0 \pm 0.1^o$. The reflected first harmonic and the generated second harmonic, making an angle of 57.7^o to the horizontal, are both oriented towards the top right of the image. It is impossible for a reflected first harmonic or second harmonic to go in any other direction, because the wall prevents this. In contrast, the arrangement for Figure 2(b) was $\theta = 25.0 \pm 0.1^o$ and $\phi = 35.0 \pm 0.1^o$, so that the reflected first harmonic is directed towards the bottom left of the image. The second harmonic beam still proceeds towards the top right, as its progress is not blocked by the wall. It was not possible to realise a third scenario, in which both beams are directed towards the lower left corner. Numerical simulations demonstrated that this is due to the fact that the energy of the generated second harmonic beam is too weak to be detected with the current experimental set-up.



(a) (b)

FIGURE 2 : Reflected internal wave beams. (a) $\theta = 25^o, \phi = 15^o$. (b) $\theta = 25^o, \phi = 35^o$.

CONCLUSIONS

We have performed two careful sets of experiments in which "synthetic schlieren" was used to obtain new results on the dynamics of rotating and reflecting internal wave beams. The results agree very well with both theory and numerical simulations, and are relevant to atmospheric and oceanic dynamics.

Bibliography

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