MORPHOLOGICAL STABILITY OF DIRECTIONAL SOLIDIFICATION UNDER TEMPERATURE MODULATIONS

C. A. Chung, K. H. Ho and W. Z. Chien

Department of Mechanical Engineering, National Central University, Jhongli, Taiwan 320

Summary We study the instability of a planar solid-melt interface during directional solidification of a binary mixture. The molten zone is oscillated by an imposed temperature modulation. The basic state is solved analytically with the bulk melt quiescent by expanding the governing equations in terms of the small amplitude of modulation. A preliminary investigation on the morphological instability of the basic state is presented.

INTRODUCTION

The possibility of processing crystal growths in space is attractive because the buoyancy-driven convection may be eliminated in such a low-level background of gravitation. However other physical factors may in turn dominate the casting conditions. For example, the g-jitter due to the inherent mechanical vibrations may itself induce buoyant convection, and the related problems have been drawn a great deal of research efforts [1]. In this paper, we consider another potential factor, the temperature fluctuations. Such a time-dependent variation may result from the orbital maneuvers; the space laboratory may periodically face the sun leading to fluctuations of the laboratory temperature. In addition earth based experiments have also shown the molten region can undergo significant temperature oscillations induced by such as the thermo-capillary instability of the floating zone [2]. We therefore study the stability of a planar solid-melt boundary during directional solidification of a binary alloy assuming the temperature is being modulated in a sinusoidal way. We hope the present study will gain an insight into the morphological instability of oscillatory molten zones.

MATHEMATICAL FORMULATION AND ASSUMPTIONS

The system considered consists of a binary alloy solidifying from below. The solid-melt interface is described by $z = h(x, y, t)$. The interface is assumed to advance into the bulk melt at a time-mean constant pulling rate V and have a planar shape before the instability occurs. We choose the coordinates fixed at the interface moving upward at the same speed V. The governing equations are the continuity equation, the momentum equation and the concentration equation [3]:

$$\nabla \cdot \boldsymbol{u} = 0\,, \quad S_c^{-1}\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial z} + \boldsymbol{u}\cdot\nabla\right)\boldsymbol{u} = -\nabla p + \nabla^2 \boldsymbol{u} + R_c C \hat{e}_z\,, \quad \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial z} + \boldsymbol{u}\cdot\nabla\right) C = \nabla^2 C\,.$$

At the solid-melt interface $z = h(x, y, t)$, the boundary conditions are

$$\boldsymbol{u}\times\boldsymbol{n} = \boldsymbol{0}\,, \quad \boldsymbol{u}\cdot\boldsymbol{n} = 0\,, \quad (k-1)C\left(1 + \frac{\partial h}{\partial t}\right)\hat{e}_z \cdot \boldsymbol{n} = \nabla C \cdot \boldsymbol{n}\,, \quad T = MC - UK$$

These equations represent respectively the no-slip condition, the no-penetration condition, the conservation of solute at the interface, and the thermodynamic relation between the melting temperature and composition. Note that the surface energy effect (the Gibbs-Thompson effect) is also included in the thermodynamic condition, where U is the dimensionless capillary length and K the curvature of the interface (assumed negative for a concave projection into the melt). The molten zone is assumed to have a time-mean constant height H, where the melt is motionless and the solute concentration is fixed at an imposed value. Therefore the corresponding boundary conditions at $z = H$ are,

$$\boldsymbol{u} = \boldsymbol{0}\,, \quad C = 1\,.$$

The governing equations and boundary conditions have been made dimensionless with the solute-field scales: V for velocity, D/V for length, (where D is the solute diffusivity), D/V^2 for time, the concentration value at $z = H$ for concentration, and the freezing temperature of the pure solvent for temperature. The two dimensionless parameters appearing in the governing equations are the Schmidt number S_c and the solutal Rayleigh number R_c. To simplify the analysis, the frozen temperature assumption is adopted by which the temperature is assumed fixed at its imposed value. We assume the temperature is modulated by

$$T = T_0(1 + \delta \cos \Omega t) + Gz\,,$$

where δ is the amplitude of the temperature oscillation, Ω is the input frequency of modulation, and G is the temperature gradient assumed to be a constant.

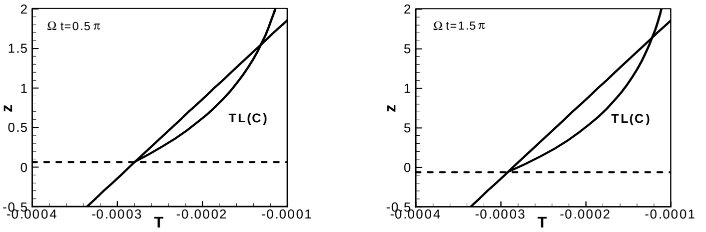

Figure 1. The instantaneous temperature T and melting temperature T_L at two phases of a period: (a) $\Omega t = 0.5\pi$, (b) $\Omega t = 1.5\pi$.

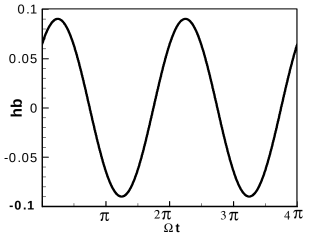

Figure 2. The interface position oscillates as a function of time.

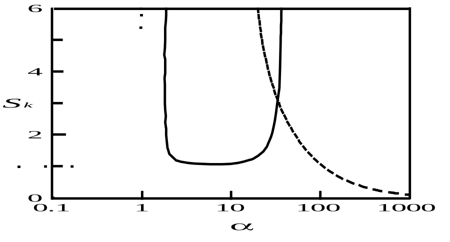

Figure 3. The neutral curves in terms of the Sekerka number S_k and the wave number α . The left curve corresponds to the synchronous mode and the right curve the subharmonic mode.

RESULTS

Basic state

Before the onset of morphological instability, the melt is quiescent relative to the laboratory frame and the solid-melt interface remains flat. We call this state the basic state. The basic state is solved by expanding the governing equations in term of δ , which is assumed to be small. The basic-state interface position and concentration have the form

$$h_b = \delta h_1(t) + \ldots$$

$$C_b = C_0(z,t) + \delta C_1(z,t) + \ldots$$

Shown in figure 1 are the instantaneous temperature T and the melting temperature $T_L(C)$ at two phases of a period: (a) $\Omega t = \pi/2$ and (b) $\Omega t = 3\pi/2$. Because the melting temperature is a function of the concentration C , the melting temperature curve implies the existence of a concentration boundary layer near the interface. The depth as well as the strength of the supercooling region varies in a sinusoidal way. The time dependence of the interface position h_b is shown in figure 2. The interface moves back and forth sinusoidally relative to the moving frame. We found the amplitude of the interface position decreases as the input frequency is increased.

Linear stability analysis

A preliminary linear stability analysis on the basic state is performed assuming $R_c = 0$. Namely, we have neglected the gravitational effects. Two morphological instability modes are found. Their neutral curves are shown in figure 3 in terms of the Sekerka number S_k and the wave number α for the case of $\Omega = 0.3$ and $\delta = 0.01$. The left curve of smaller values of the wave numbers is the synchronous mode having the same oscillatory frequency as that of the input temperature modulation. The right curve is the subharmonic mode that has the frequency half the input frequency. As shown, the subharmonic mode is likely to be more critical and dominant.

CONCLUSIONS

We consider in this paper the directional solidification of a binary mixture undergoing temperature modulations. The solid-melt interface moves back and forth in a sinusoidal way. The interfacial instability tends to occur with a subharmonic mode whose oscillatory frequency is half the input frequency. The wavelengths of the subharmonic mode are much smaller than that of the synchronous mode.

References

[1] Murray, B. T., Coriell, S. R., McFadden, G. B.: The Effect of Gravity Modulation on Solutal Convection during Directional Solidification. *J. Crystal Growth.* 110: 713-723, 1991.

[2] Hu, C. C., Chen, J. C., Huang, C. H.: Effect of Pulling Rates on the quality of YIG Single Crystal Fibers. *J. Crystal Growth.* 225: 257-263, 2001.

[3] Chung, C. A., Chen, F.: Instability in Directional Solidification under Inclined Rotation. *J. Fluid Mech.* **477**: 381-408, 2003.