XXI ICTAM, 15–21 August 2004, Warsaw, Poland

A NEW APPROACH FOR THE FE MODELLING OF COHESIVE CRACKS

Julia Mergheim, Ellen Kuhl, Paul Steinmann
Chair of Applied Mechanics, Department of Mechanical Engineering, University of Kaiserslautern, PO-Box: 3049, 67653 Kaiserslautern, Germany

Summary The present contribution is concerned with the computational modelling of cohesive cracks, whereby the discontinuity is not limited to interelement boundaries, but is allowed to propagate freely through the elements. Inelastic material behaviour is described by discrete constitutive models, formulated in terms of tractions and displacements at the surface. Details on the implementation and numerical examples are given.

INTRODUCTION

The modelling of propagating discontinuities was recently considered by several authors, following different approaches. As an approved method, most similar to the present one, we mention the partition of unity method, which was introduced in [1] and [2] and was applied for the modelling of cohesive cracks, for example, in [3]. In the present work we will follow the approach recently suggested in [4]. Thereby neither the jump nor the strain jump is an explicit variable, but it is only worked with polynomial approximations and displacement degrees of freedom. In the next section the governing equations and the weak formulation are recapitulated, afterwards the idea of constructing elements with internal discontinuities is highlighted. Then some details about the applied discrete constitutive law and the implementation are given and the performance of the method is pointed out by means of numerical examples.

WEAK FORMULATION

Let Ω denote the configuration occupied by an initially linear elastic body. The boundary $\partial\Omega$ with the outward normal vector $\boldsymbol{n}_e$ is subdivided in $\partial\Omega = \Gamma_N \cup \Gamma_D$ with $\Gamma_N \cap \Gamma_D = \emptyset$, where either Neumann or Dirichlet boundary conditions are prescribed. Furthermore Ω exhibits a smooth internal boundary Γ_I, which divides Ω into the parts Ω_1 and Ω_2. The unit normal vector $\boldsymbol{n}$, associated with Γ_I, points from Ω_2 to Ω_1. The jump in the displacement field $\boldsymbol{u}$ along Γ_I is defined by $[\![\boldsymbol{u}]\!] = \boldsymbol{u}_1 - \boldsymbol{u}_2$. The equilibrium equation and boundary conditions read:

$$-\boldsymbol{\nabla}\cdot\boldsymbol{\sigma} = \boldsymbol{b} \text{ in } \Omega, \qquad \boldsymbol{u} = \bar{\boldsymbol{u}} \text{ on } \Gamma_D, \qquad \boldsymbol{\sigma}\cdot\boldsymbol{n}_e = \bar{\boldsymbol{t}} \text{ on } \Gamma_N, \qquad \boldsymbol{\sigma}\cdot\boldsymbol{n} = \boldsymbol{t}([\![\boldsymbol{u}]\!]) \text{ on } \Gamma_I, \tag{1}$$

whereby $\bar{\boldsymbol{t}}$ and $\bar{\boldsymbol{u}}$ are the prescribed tractions and the prescribed displacements, $\boldsymbol{\sigma}$ is the Cauchy stress and $\boldsymbol{b}$ is the body force. For the considered geometrically linear formulation the symmetric strain tensor $\boldsymbol{\epsilon}$ is given by the kinematic equation $\boldsymbol{\epsilon} = \frac{1}{2}[\nabla\boldsymbol{u} + \nabla^t\boldsymbol{u}]$. The constitutive relation in the bulk is given by $\boldsymbol{\sigma} = \boldsymbol{D} : \boldsymbol{\epsilon}$, whereby $\boldsymbol{D}$ is the fourth order constitutive tensor. The description of the inelastic behaviour of the material is completely covered by the discrete constitutive model applied at the internal boundary Γ_I. The weak formulation, which is obtained by multiplication of (1) with a test function $\delta\boldsymbol{u}$ and integrating by parts, possesses an additional contribution due to the tractions $\boldsymbol{t}$ along Γ_I

$$\int\limits_{\Omega_1\cup\Omega_2} \delta\boldsymbol{\epsilon} : \boldsymbol{\sigma}(\boldsymbol{u})\, dV + \int\limits_{\Gamma_I} [\![\delta\boldsymbol{u}]\!]\cdot\boldsymbol{t}([\![\boldsymbol{u}]\!])\, dA = \int\limits_{\Omega_1\cup\Omega_2} \delta\boldsymbol{u}\cdot\boldsymbol{b}\, dV + \int\limits_{\Gamma_N} \delta\boldsymbol{u}\cdot\bar{\boldsymbol{t}}\, dA. \tag{2}$$

Thereby the traction vector $\boldsymbol{t}$ depends on the discrete constitutive law in terms of the displacement jump.

DISCRETIZATION

In this section details about the discretization are given. The weak form (2) will be solved, using finite elements which allow for a discontinuity intersecting the elements, following the approach suggested in [4].
To construct an element E with an internal discontinuity, we consider that E is divided by Γ_I into $E_1 := \Omega_1 \cap E$ and $E_2 := \Omega_2 \cap E$. The displacement field $\boldsymbol{u}$ will be continuous on each part, but discontinuous over Γ_I, therefore $\boldsymbol{u}$ can be represented by

$$\boldsymbol{u} = \begin{cases} \boldsymbol{u}_1 & \text{in} \quad E_1 \\ \boldsymbol{u}_2 & \text{in} \quad E_2 \end{cases} \tag{3}$$

To approximate the function $\boldsymbol{u}_1$ on E_1 one needs the usual number of degrees of freedom, depending on the desired polynomial degree. Even though $\boldsymbol{u}_1$ is only defined on E_1, it can be represented by its nodal values at the existing nodes of the element E and the standard basis functions. The same applies for function $\boldsymbol{u}_2$, which has, due to the discontinuous characteristic of $\boldsymbol{u}$, no relation to $\boldsymbol{u}_1$. That means that for an element intersected by a discontinuity two independent copies of the standard basis functions N are used, whereby one set is put to zero on one side of the discontinuity, while it takes its usual values on the opposite side, and vice versa:

$$N_1^j = \begin{cases} N^j & \text{in} \quad E_1 \\ 0 & \text{in} \quad E_2, \end{cases} \qquad \text{and} \qquad N_2^j = \begin{cases} 0 & \text{in} \quad E_1 \\ N^j & \text{in} \quad E_2. \end{cases} \tag{4}$$

It is recognizable that this set of basis functions with an internal discontinuity can be easily constructed for any standard finite element in 2D or 3D. The additional degrees of freedom are introduced at the existing nodes and the points of intersection between element edges and the interface as well as the geometry of the element parts are only needed for the evaluation of the weak form.

DISCRETE CONSTITUTIVE MODEL

In the proposed method discrete constitutive models are applied to model the inelastic material behaviour. The discrete models are formulated in terms of tractions and displacements and they are applicable at the internal surface. In the following a discrete damage-type model for quasi-brittle materials is introduced, with the tensile stress f_t and the fracture energy G_f being the main parameters. An exponential softening in normal direction and a constant shear stiffness in tangential direction is assumed. The traction vector $\boldsymbol{t}$ is determined by

$$t_n = f_t \exp\left(-\frac{f_t}{G_f} [\![u_n]\!]\right), \qquad t_m = d\,[\![u_m]\!] \qquad \text{and} \qquad \boldsymbol{t} = t_n\,\boldsymbol{n} + t_m\,\boldsymbol{m}, \tag{5}$$

whereby d is the shear stiffness and $\boldsymbol{m}$ is the tangential vector associated to Γ_I. This discrete constitutive model is chosen because of its simplicity with respect to the implementation. Due to the constant shear stiffness, which is a valid assumption for mode 1 failure only, the tangent stiffness matrix preserves its symmetry. Nevertheless the introduction of a more general constitutive model is straightforward.

IMPLEMENTATION AND NUMERICAL EXAMPLE

To describe a propagating discontinuity we need to propose a failure criterion, a method to determine the alignment of the discontinuity and an adequate integration scheme for the intersected elements. During the calculation the principle stresses in the element ahead of the discontinuity tip are monitored. If the stresses exceed the tensile strength, the discontinuity is introduced as a straight line through the element and is enforced to be geometrically continuous. To determine the right direction of the extension of the discontinuity, non-local stresses are calculated. The non-local stress tensor is computed as the weighted average of the stresses at the gausspoints within an interaction radius around the tip. The discontinuity is extended in the direction perpendicular to the non-local principle stress.

In the elements, intersected by a discontinuity, the same basis functions as in the ordinary elements are used. But since the geometry of the element parts varies, the element fragments are triangulated into sub-domains and standard Gauss quadrature is applied. Additional integration point are inserted along the internal boundary Γ_I, to integrate the contributions depending on the traction vector.

To show that the proposed method allows for the propagation of a discontinuity, independent of the mesh structure, two numerical examples are given. We consider a three-point bending test, whereby a simply supported beam is loaded by an imposed displacement at the center of the top edge. The analysis is performed with linear, three-noded triangles, a full Newton-Raphson solution procedure is used and linear elastic behaviour of the continuum is assumed. Figure (1) shows the propagation of a discontinuity introduced at the center of the beam and figure (2) pictures a discontinuity which is introduced excentric. Both the simpler case of the straight crack and the curved crack are calculated independently of the mesh structure.

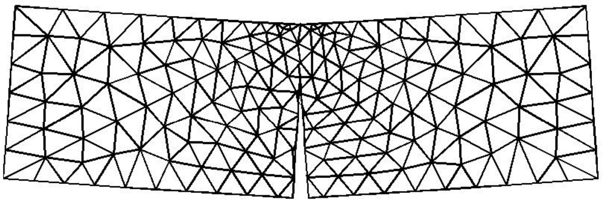

Figure 1: Propagation of centered crack

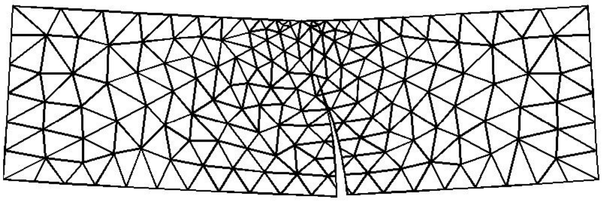

Figure 2: Propagation of excentered crack

CONCLUSIONS

We introduced a finite element method for the modelling of cohesive cracks. The characteristic feature of the method is the construction of the elements with an internal discontinuity, which is independent of the element type and uses only the standard basis functions. The method was applied to model cohesive cracks, making use of a discrete constitutive law. The presented numerical examples point out that the method allows for simulating propagating discontinuities independent of the mesh structure.

References

[1] Belytschko T, Black T. Elastic crack growth in finite elements with minimal remeshing. *International Journal of Numerical Methods in Engineering* 1999; **45**(5): 601-620.

[2] Moës N, Dolbow, J, Belytschko T. A finite element method for crack growth without remeshing. *International Journal of Numerical Methods in Engineering* 1999; **46**(1): 131-150.

[3] Wells GN, Sluys LJ. A new method for modelling cohesive cracks using finite elements. *International Journal of Numerical Methods in Engineering* 2001; **50**(12): 2667-2682.

[4] Hansbo A, Hansbo P. A finite element method for the simulation of strong and weak discontinuities in elasticity. *Computer Methods in Applied Mechanics and Engineering*; to appear.