REANALYSIS OF AN SEA HIGH-FREQUENCY VIBRATION CALCULATION BASED ON THE VTCR

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INTRODUCTION

The analysis of the vibration behavior of structures in the high-frequency range has become a key issue for engineers involved in structural design. In order to control and reduce displacements and stresses, the dynamic response must be properly and accurately predicted.
The finite element method (FEM [1]) is frequently used to analyze the structural response of complex structures. However, the analysis of the response of a system becomes more difficult as the excitation frequency increases, and this method is limited to the low frequency range ([2]). The difficulty resides in the length of variation of the phenomena being studied, which diminishes as the frequency increases. In the contest of the FEM, one often considers that about eight elements must be used to represent each structural wavelength adequately ([3]); thus, high-frequency calculations would require an unreasonably large number of degrees of freedom.
Because of the difficulties involved in developing a detailed model for high-frequency vibrations, considerable research effort has been devoted to the development of more appropriate techniques. One of the most commonly used techniques for high-frequency vibrations is Statistical Energy Analysis (SEA [4]). This procedure consists in modeling the system as a set of subsystems, each of which is assigned a single response variable corresponding to its vibrational energy. Using a description of the energy exchanged among the subsystems, this method yields the dynamic response of each subsystem averaged in time and in space. However, the SEA cannot provide local information on the response in each subsystem, since this response is characterized only by a single energy level. Therefore, an improvement of the SEA model would be welcome. This is the aim of this presentation.
The method of calculation of the vibrations of slightly damped elastic structures in the high-frequency range which is presented here is based on the reanalysis of global information to derive more local information. The global information is given by the SEA. The local information is obtained by reanalyzing the SEA data using the Variational Theory of Complex Rays (VTCR [5]). This method can be viewed as a refined calculation (VTCR analysis) of a large-scale analysis (SEA). It is based on a "Saint Venant" energy principle which gives the spatial distribution of energy density at high frequency, thus yielding a description of the system's behavior which is more accurate than the average constant value given by SEA. Initial applications dealing with simple rod and plate systems are presented and show that the method produces good effective local results under high-frequency exctitation.

THE VTCR

The VTCR is a method dealing with medium-frequency vibrations. The first feature which characterizes the VTCR is the use of a new variational formulation of the problem to be solved, which was developed in order to allow within the substructures approximations which are *a priori* independent, which means that they are not required to verify *a priori* the transmission conditions at the interfaces between substructures, whether in terms of displacements or stresses. Instead, these conditions are included in the variational formulation. The second feature defining the VTCR is the introduction of two-scale approximations with a strong mechanical meaning: the solution is assumed to be properly described as the superposition of an infinite number of local propagative and evanescent waves. These basic waves (which can be interior propagative waves, edge evanescent waves or corner evanescent waves) verify the laws of dynamics. All wave directions are taken into account. The unknowns are the amplitudes of these waves, whose wavelengths are relatively large. The discretization of these amplitudes leads to the resolution of a matrix system whose size is small compared to that of an FEM model, since only large-wavelength quantities are being used.
The example of Figure 1 shows the comparison between the solutions given by the VTCR (left) and by NASTRAN (right) for a missile's structure. The two results are similar. The NASTRAN solution (FEM calculation) was obtained using 10 nodes per wavelength, which leads to $28,800$ degrees of freedom (dofs). The VTCR solution was obtained with 672 dofs.

THE REANALYSIS STRATEGY

The reanalysis strategy proposed is based on the calculation of the local energy density distribution in two steps: first, a large-scale calculation, then a refined calculation. The large-scale resolution consists in an SEA calculation over the whole structure divided into many substructures, which yields the average density of vibration energy for each substructure. The refined solution consists in a VTCR calculation over each substructure, which yields the distribution of the average energy density over the substructure.

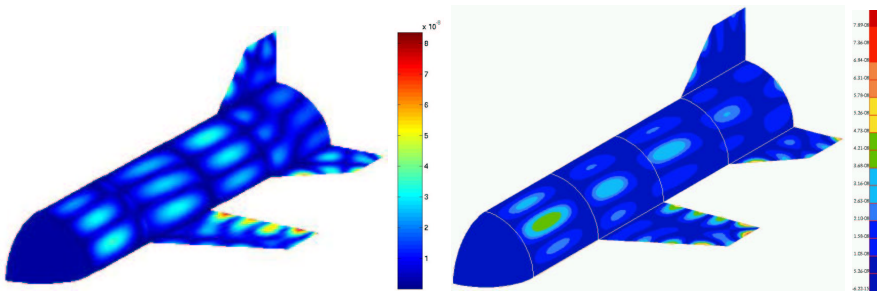

Figure 1: Comparison between the VTCR (left, 672 dofs) and NASTRAN (right, 28, 800 dofs). The scales are the same

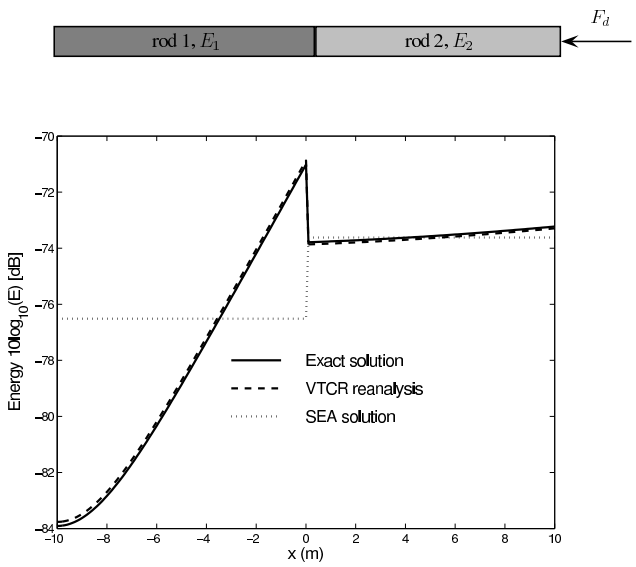

Figure 2: Reanalysis method applied to a rod problem

The large-scale resolution by the SEA requires a knowledge of the injected power and of the coupling loss factors, which can be evaluated using standard methods ([6]). This first calculation provides the power that flows from one substructure to another. This power is assumed to be correct.
The refined solution in each substructure calculated using the VTCR is based on the variational formulation of the vibration problem defined in each substructure of the whole structure. The substructure's boundary conditions are prescribed forces or displacements such that the power injected along each edge is that calculated by the SEA. The local displacements of this substructure subjected to these prescribed conditions are calculated using the VTCR; from this calculated solution, one retains only the total average (in time and space) energy density distribution, which is a quantity whose distribution does not depend on the way the power is injected, but only on the magnitude of the power injected along each edge. This last property, which can be viewed as a "Saint Venant" energy principle, has been proven for rod, the beam and plate structures.
An example of this approach is given in Figure 2, which represents a structure composed of two rods with different Young's moduli. Three curves are shown: the exact calculation (continuous line), the SEA solution (step function per rod) and the SEA/VCTR reanalysis solution (dashed line). The reanalysis solution is very close to the exact solution.

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