APPLICATION OF THE CONTINUUM MECHANICS IN THE TEXTILE FABRICS

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Summary Textile fabrics represent complicated structures. The mechanical properties of these formations are possible to describe on the basis of theirs specific idealization. One of the methods of problem solving of the mechanics textiles is substitute of the textile formation by continuous environment – continuum. The substitute continuum has identical mechanical properties like testing textile fabric.

The substitution of the loaded textile fabric with specific structure by the continuum with identical mechanical properties enables to use the equations of the continuum mechanics. Then on the basis of these equations we can define the basic mechanical fabric properties. Dependencies between Euler and Lagrange coordinates of the points is possible to determine on the basis of the measured shifts of the monitoring points of the textile fabric, see Fig. 1:

$x_i^p = \overset{\circ}{x}{}_i^{p} + u_i^p$, i = 1,2; p… is the number of point. (1)

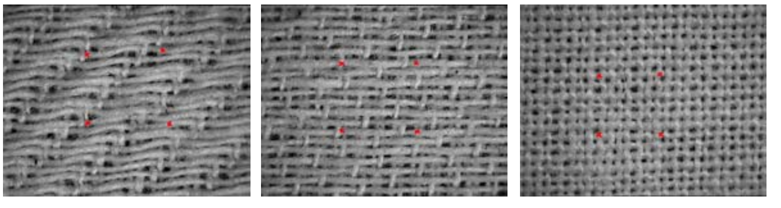

Fig. 1 Depiction of the monitoring points of the textile fabric with different thread's interlacing

For description of mechanical properties in this case we needn't thing the fictive thickness of the textile fabric if we use the definition of the relative force [Nm^{-1}] influences in area of the textile fabric. Therefore for description of the mechanical properties of the fabric we will use not tensors in the space (three-dimensional) but the tensors on the surface (two-dimensional). The tensors are:

- the material deformation gradient $F = \left(x_{i;j}\right)$, (2)
- Cauchy tensor of relative forces $\sum = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}$, (3)
- this tensor comes from Cauchy equation $S_i = \sigma_{ij} n_j$ at the element of textile;
- the tensor of extension $U = \left(F^T F\right)^{1/2}$; (4)
- the tensor of deformation $E = \frac{1}{m}\left(U^m - I\right)$ for $m = -2, -1, 0, 1, 2$; (5)
- the tensor of the relative forces $S = \begin{pmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{pmatrix}$, (6)
- the tensors satisfy the condition of the conjugation with tensors of deformation E;
- tensor of rotation R is given from the tensor F by the polar decomposition.

The measures of the deformation as well as tension are different. For calculation of the modulus of elasticity is necessary to choose suitable combination of the couple of the deformation tensor and tension tensor. For description of the textile fabric, from under mentioned conjugated couple are most used the couple with number III and IV:

I. $m = 2; \quad E = \frac{1}{2}\left(F^T F - I\right) \Leftrightarrow JF^{-1} \sum \left(F^{-1}\right)^T = S,$ (7)

II. $m = -2; \quad E = \frac{1}{2}\left(I - \left(F^T F\right)^{-1}\right) \Leftrightarrow JF^T \sum F = S,$ (8)

III. $m = 0; \quad E = ln\, U \Leftrightarrow JR^T \sum R = S,$ (9)

IV. $m = 1; \quad E = U - I \Leftrightarrow \frac{J}{2}\left[F^{-1} \sum R + R^T \sum \left(F^{-1}\right)^T\right] = S,$ (10)

V. $m = -1; \quad E = I - U^{-1} \Leftrightarrow \frac{J}{2}\left[F^T \sum R + R^T \sum F\right] = S.$ (11)

The loaded textile fabrics have anisotropic character. In dependence at the manner of the loading (the loading in the principal axes of the textile structure or the loading in the general angle in view of principal axes of the textile structure) the textile fabrics can have the character of:

1) monoclinic anisotropy with six modulus of elasticity $\overline{E}_{ij}$ in the surface of the textile fabric,
2) rhombic anisotropy with four modulus of elasticity in the surface of the textile fabric.

If the tensors satisfy the specific conditions [1] for Poisson number $\overline{\nu}_{12}$, then the textile fabrics have the character of:

3) hexagonal anisotropy with two modulus of elasticity at the loading in the principal axes of the textile structure,
4) tetragonal anisotropy with four modulus of elasticity at the loading in the general direction.

The lane above the quantities $\overline{E}_{ij}$, $\overline{\nu}_{12}$ represent the parameters of the plane state of stress. For description of the modulus of elasticity $\overline{E}_{ij}$ is necessary to resolve the non-linear problem of the continuum mechanics using the constitutive equations:

$$Jc_{ij} = 2\frac{\partial\Phi}{\partial\Psi_k}\frac{\partial\Psi_k}{\partial\varepsilon_{ij}} \quad i, j = 1, 2; k = 1, 2, \ldots, n, \tag{12}$$

where:

c_{ij}, ε_{ij} are the components of the tensors of the selected conjugated couple,

2Φ is the elastic potential given by the conjugated couple and Ψ_k its invariant.

According to type of anisotropy above-mentioned equations is necessary to connect with under mentioned dependencies:

$$tg2\omega = \frac{2\left(\overline{E}_{14} + \overline{E}_{24}\right)}{\overline{E}_{11} - \overline{E}_{22}}, \tag{13}$$

$$\overline{E}_4 = \frac{\tau_i}{\gamma_i}, \tag{14}$$

$$\overline{E}_{11}\overline{E}_{22} - \overline{E}_{12}^2 - \overline{E}_4\left(\overline{E}_{11} + \overline{E} +_{22} + 2\overline{E}_{12}\right) = 0, \tag{15}$$

$$2\overline{E}_4 - \overline{E}_{11} + \overline{E}_{12} = 0. \tag{15a}$$

where:

ω is the angle between the principal axis of anisotropy and the direction of the textile loading,

τ_i, γ_i are the intensity of relative shearing forces and shear strain, the equation (15a) describes the transversal isotropy of the textile.

Discussion

Firstly –Description of the textile mechanics

Identification of the mechanical textile fabric properties stressed by uniaxial or biaxial state of stress is possible to pursue only for specific concrete state of stress and strain. The changes of the state of stress lead to the change of the mechanical parameters.

Secondly –Description of the textile structure

Each of textile fabric has different structure; it is not homogeneous [2]. From the textile parameters viewpoint is necessary to describe the structure of the textile as well as the kind of textile fabric (woven fabric, non-woven fabric or knitting fabric). The input parameters of these fabrics and their parameters in the longitudinal as well as transverse section influence the final mechanical properties [3].

References

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