DYNAMICS OF PERTURBATIONS AND SHEAR BAND INSTABILITIES

Davide Bigoni[*] **Domenico Capuani**[**]
[*]*Dipartimento di Ingegneria Meccanica e Strutturale, Università di Trento, Via Mesiano 77 – 38050 Povo, Trento, Italia. Email* bigoni@ing.unitn.it.
[**] *Dipartimento di Ingegneria, Università di Ferrara, Via Saragat 1– 44100 Ferrara, Italia. Email* cpd@dns.unife.it

Summary The dynamic Green's function is obtained for an infinite nonlinear-elastic, incompressible solid homogeneously deformed and subject to time-harmonic incremental perturbations. The formulation is employed to investigate the effects of perturbations on the dynamic behaviour of solids near the shear band formation threshold. Results shed light on the problem of interaction between dynamics and shear band formation.

THE GREEN'S FUNCTION

Superimposed small disturbances are considered under dynamic (time-harmonic) conditions upon an elastic, incompressible, infinite body described by the Biot (1965) incremental equations and subject to plane, homogeneous –but arbitrary– deformation. Following Willis (1991), the dynamic, infinite body Green's function (or fundamental solution) is found for this problem. This solution –which paves the way for integral methods in incremental elastodynamics, generalizing previous findings by Brun et al. (2003)– is employed to analyze shear band formation following a perturbative approach.

THE PERTURBATIVE APPROACH

The perturbative approach is a generalization of that developed by Bigoni and Capuani (2002) for quasi-static deformation [see also the related results by Radi et al. (2002)] and consists in the analysis of the incremental displacement maps produced by a dipole formed by two unit, opposite forces. In the quasi-static case, it has been shown that a perturbation close enough to the elliptic boundary may induce localization patterns that are formally excluded in terms of the usual bifurcation approach (e.g. Rice, 1977). For instance, a localized pattern is clearly visible in Fig. 1 pertaining to a Mooney-Rivlin material, pre-deformed up to a stretch level close to the value 3, at which shear bands are formally excluded in the framework of the bifurcation approach (see Bigoni and Capuani, 2002, for details).

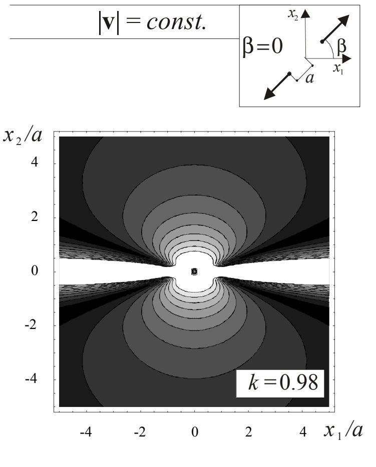

Fig. 1- Level sets of the modulus of incremental displacement vector for Mooney-Rivlin material for a dipole of width a, aligned along the x_1 axis (i.e. $\beta = 0$). Pre-stress parameter $k = 0.98$ corresponds to a stretch λ close to 3.

In this work, maps of incremental displacement produced in an infinite body by a harmonically pulsating dipole are analyzed with the aim of investigating the response to dynamic perturbations of a solid near the boundary of shear band formation. The solution now depends on the frequency of the applied perturbation, which turns out to represent an important parameter in the analysis.

References

[1] Bigoni, D., Capuani, D., 2002. Green's function for incremental nonlinear elasticity: shear bands and boundary integral formulation. J. Mech. Phys. Solids 50, 471-500.
[2] Biot, M.A., 1965. Mechanics of Incremental Deformations. J. Wiley and Sons, New York.
[3] Brun, M., Capuani, D., Bigoni, D. A boundary element technique for incremental, nonlinear elasticity. Part. I: Formulation and Part. II: Bifurcation and shear bands. *Comp. Meth. Appl. Mech. Eng.* 192 (22-24), 2003, 2461-2479.
[4] Radi, E., Bigoni, D., Capuani, D. 2002 Effects of pre-stress on crack-tip fields in elastic, incompressible solids. *Int. J. Solids Structures*, 39, 3971-3996.
[5] Rice, J. R., 1977. The localization of plastic deformation. In: Theoretical and Applied Mechanics (ed. W.T. Koiter), pp. 207-220. North-Holland, Amsterdam.
[6] Willis, J.R., 1991. Inclusions and cracks in constrained anisotropic media. In: Wu, J.J., Ting, T.C.T. and Barnett, D.M. (Eds.), Modern Theory of Anisotropic Elasticity and Applications. SIAM, Philadelphia, pp. 87-102.