CONTROL OF RANDOM DYNAMICS OF A RIGID ROCKING BLOCK

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Summary. The problem of minimizing the toppling probability of a rocking block is considered as a control problem for a stochastic discontinuous oscillator. Formally, the control task is to minimize the probability of escape the system from the domain enclosed by the unperturbed separatrix. The problem is resolved by the dynamic programming method, with application of the stochastic averaging procedure.

PROBLEM FORMULATION

The paper studies controlled dynamics of a rigid block and a block structure. Rocking oscillations are excited due to random horizontal ground motion. Assuming rigid foundation, large friction to prevent sliding, and the Newton restitution law during the impact, the only possible response mechanism under ground excitation is the rocking about the corners of the block (Fig. 1 a, b). The control task is to minimize the toppling probability with due regard to control constraints.

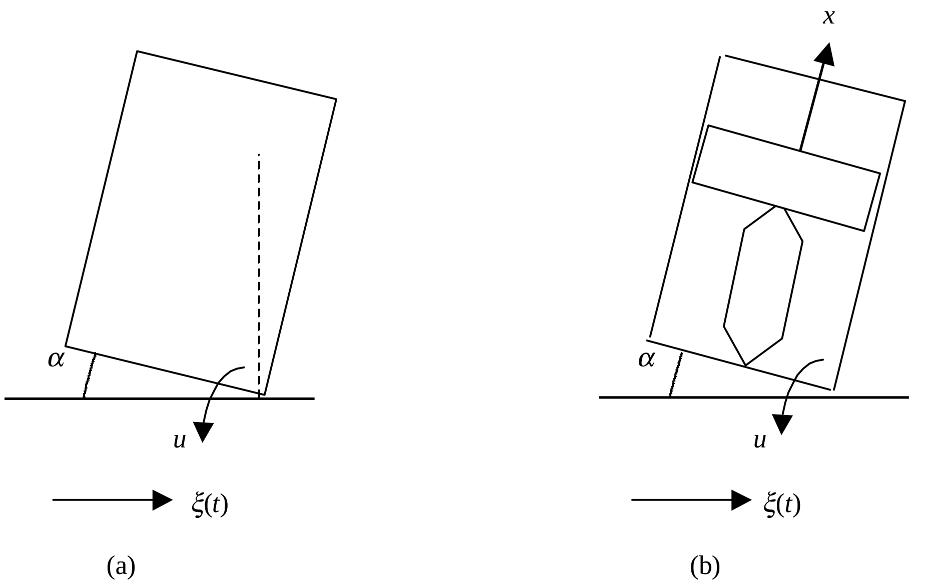

Fig.1. Rocking motion of a rigid block (a) and a block structure; α - inclination angle, u – control torque, x – mass displacement, $\xi(t)$ – ground acceleration

The response analysis of random rocking motion of a rigid block has been pursued for last decades; see [1] for the detailed review and discussion. The engineering approach for the toppling prevention lies in making use of the elastic tie-down rods or anchoring, see, e.g. [2], [3]. This paper considers the problem of the toppling prevention as a control problem.

ASYMPTOTIC ANALYSIS AND NEARLY OPTIMAL CONTROL

The first question addressed in the paper is the control problem for a rigid block. The equation of rocking motion corresponds to the equation of a quasiconservative nonlinear discontinuous oscillator. The phase plane of the conservative counterpart of the system is divided into two domains, the domain of bounded oscillations (librations) and the domain of unbounded motion (toppling), separated by the unperturbed separatrix. Escape through the separatrix is considered as a dangerous event to be avoided. The control task is thus to keep the system within the domain enclosed by the unperturbed separatrix.

The well-developed analysis and control procedure for systems with impact [4] cannot be directly applied to the strongly nonlinear system. We extend the averaging procedure to nonlinear discontinuous system. The equation of

motion is reduced to the standard form for the quasiconservative system, with the slow energy evolution and the associated fast phase. The Newton restitution law allows introduction of the impact dissipation in the equation of the slow variable. The coefficient takes the form of the periodic Dirac δ- function and allows averaging [4].
In order to simplify the system's structure and to reduce it to the form of the stochastic Ito equation, we employ the partial stochastic averaging approach [4]. At the first stage, the stochastic averaging reduces the perturbed system to the form of the stochastic Ito equation with slow and fast variables. At the second stage, the partial averaging of the deterministic coefficients, except for control, excludes fast components of the periodic coefficients and smoothes out the discontinuities in the equation of the slow motion.
The dynamic programming equation for the partially averaged system determines nearly optimal control. This equation is resolved asymptotically, by making use of the averaging in fast variable [4].
The second part of the paper centres on the effect of the secondary structure on the rocking dynamics (Fig. 1b). Motion can be presented as the superposition of the tracking rocking motion of the system as a whole and the relative oscillations of the secondary flexible structure. For a number of practical applications, the relative oscillations are small compared to the rigid motion. This allows simplification of the equations of motion and application of the asymptotic decomposition procedures to the analysis and control problems. The goal is to estimate the effect of the secondary structure to the statistical properties of motion and the probability distribution of the toppling time.

References

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