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PARTICLES LOCATED ON A PLANAR FREE-SURFACE – HYDRODYNAMIC INTERACTIONS IN QUASI-TWO-DIMENSIONAL SYSTEM

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Summary The friction and mobility matrices are constructed for a quasi-two-dimensional system consisting of spherical particles, which are suspended in a quiescent fluid and which touch a planar free surface. For such a system, the one-sphere resistance operator is calculated numerically. Moreover, the long-distance leading terms of the two-sphere mobility matrix are evaluated up to the order of $1/R^3$, where R is the interparticle distance.

INTRODUCTION

The dynamics of mesoscopically large colloidal particles suspended in a viscous fluid is strongly influenced, in addition to direct interparticle forces, by fluid-mediated hydrodynamics interactions [1]. In the past, research was mainly focused on the microhydrodynamics of three-dimensional bulk dispersions of particles. Meanwhile the center of interest has shifted to properties of colloids near boundaries like a single hard wall, colloids close to a clean or surfactant-covered fluid-fluid interface. In such systems, the particle dynamics is more complicated in comparison to an unbounded bulk suspension, since additional fluid boundary conditions have to be satisfied. Of particular interest are so-called quasi-two-dimensional dispersions, where the particles are confined to move along liquid-gas interfaces [2, 3]. A well-studied example of such a dispersion is given by micron-sized super-paramagnetic colloidal spheres suspended in water next to a water-air interface [4, 5, 6, 7]. By means of an experimental hanging-drop geometry, the spheres are gravitationally confined to lateral motion along the interface. For a quantitative description of particle diffusion in this magnetic model system, it is necessary to know precisely the lateral hydrodynamic mobility tensors of the spheres in presence of the liquid-gas interface. The mobility tensors are thus an essential input in theoretical and Brownian dynamics simulation studies of in-plane diffusion and microstructure.

In this work, we provide a general method to determine the low-Reynolds-number hydrodynamic interactions for such quasi-two-dimensional systems. In particular, we show how to construct the friction and mobility tensors for particles moving along a planar free surface. We give explicit numerical results for the single sphere mobilities in such a system. Finally, we evaluate the long-distance leading terms of the two-sphere quasi-two-dimensional mobility matrix, up to the order of $1/R^3$, where R is the interparticle distance.

THEORETICAL METHOD

The fluid in the half-space is described by the Stokes equations [1], with the free-surface boundary conditions at the planar interface and the stick boundary conditions at the surfaces of the spheres immersed in the fluid. To solve them, the corresponding Green operator, constructed by the method of images is used [8], and expansion in irreducible multipole functions [9] is applied. The mobility matrix is constructed as the multiple scattering series [10].

The essential contribution of this work is the reformulation of the scheme outlined above [8] in the special context of spheres, which touch the interface. In this case, to treat properly the lubrication effects and to avoid a spurious divergence, the symmetry of the system (inherent to the free-surface condition at the planar boundary) has been included into the new multipole functions. To evaluate them explicitly, the displacement theorems from Ref. [11] have been used.

RESULTS

Due to the lubrication forces, a sphere at the interface has only three degrees of freedom: translation along the plane and rotation around the axis perpendicular to it. Therefore the quasi-two-dimensional N-particle mobility is a 3Nx3N tensor, which determines the particle translational and angular velocities, $U_{ix}, U_{iy}, \Omega_{iz}$, for given forces and torques, $\mathcal{F}_{ix}, \mathcal{F}_{iy}, \mathcal{T}_{iz}$, with i=1,...,N. The coordinate system shown in Fig. 1, with z-axis perpendicular to the free surface, and x-axis and y-axis parallel to it.

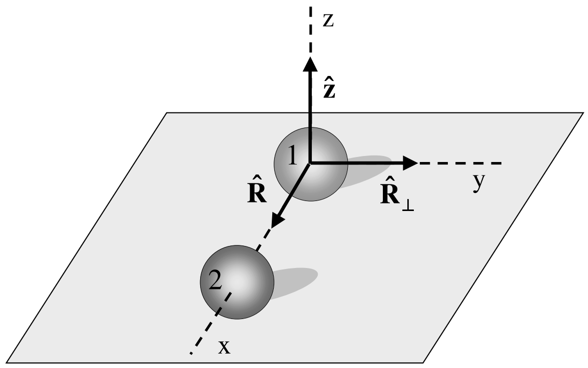

Figure 1. The system of coordinates used to describe the particles at the planar interface.

Single sphere mobilities

The quasi-two-dimensional one-sphere mobility tensor is given as

$$\boldsymbol{\mu}_F = \begin{pmatrix} \mu_F^{tt} & 0 & 0 \\ 0 & \mu_F^{tt} & 0 \\ 0 & 0 & \mu_F^{rr} \end{pmatrix}, \tag{1}$$

with $\mu_F^{tt} = 1.3799554\,/\,(6\pi\eta a)$ and $\mu_F^{rr} = 1.10920983\,/\,(8\pi\eta a^3)$.
These coefficients are equal to the inverse of the drag and turn coefficients for two touching spheres, calculated in Ref. [12] as 1.3801 and $4\,/\,\zeta(3)\,/\,3 = 1.1092098301...$, respectively.
The method developed in this work allows to evaluate numerically with a high precision also other coefficients, which determine the reaction of a single sphere on an external ambient fluid flow.

Two-sphere mobility tensor for large interparticle distance

The quasi-two-dimensional mobility tensor for two spheres is determined by 3x3 Cartesian tensors $\bar{\boldsymbol{\mu}}_{11}$ and $\bar{\boldsymbol{\mu}}_{12}$, which are specified by the relation,

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \Omega_{1z} \end{pmatrix} = \bar{\boldsymbol{\mu}}_{11} \begin{pmatrix} \mathcal{F}_{1x} \\ \mathcal{F}_{1y} \\ \mathcal{T}_{1z} \end{pmatrix} + \bar{\boldsymbol{\mu}}_{12} \begin{pmatrix} \mathcal{F}_{2x} \\ \mathcal{F}_{2y} \\ \mathcal{T}_{2z} \end{pmatrix}. \tag{2}$$

The long-distance leading terms of the mobility tensor are given as $\bar{\boldsymbol{\mu}}_{11} = \boldsymbol{\mu}_F + o(1/R^3)$ and

$$\bar{\boldsymbol{\mu}}_{12} = \mu_0^{tt}\left[\frac{1}{R}\begin{pmatrix} \frac{3}{2} & 0 & 0 \\ 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{1}{R^3}\begin{pmatrix} 1.159862... & 0 & 0 \\ 0 & 0.111686... & 0 \\ 0 & 0 & 0 \end{pmatrix}\right] + \frac{\mu_0^{rt}}{4R^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} - \frac{\mu_0^{rr}}{8R^3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ + \quad o(1/R^3), \tag{3}$$

where R is the dimensionless interparticle distance, normalized by the particle diameter. Note that x-axis is along versor $\hat{\mathbf{R}}$, parallel to the line of sphere centers, and y-axis is along the perpendicular versor $\hat{\mathbf{R}}_\perp$, as depicted in Fig. 1.

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