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MODELING OF VISCOPLASTIC CONSTITUTIVE EQUATION FOR POLYMERS BY TAKING INTO ACCOUNT STRAIN RECOVERY

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Summary A viscoplastic constitutive equation for inelastic deformation of polymers is formulated by combining the kinematic hardening creep theory of Malinin and Khadjinsky with the nonlinear kinematic hardening rule of Armstrong and Frederick. The nonlinear kinematic hardening rule is modified in order to describe peculiar strain recovery of polymers during unloading in particular. Experimental results for polyethylene are simulated by the constitutive equation and the validity of the modification is verified.

INTRODUCTION

Polymers reveal significant viscoplastic deformation at the room temperature. Peculiar strain recovery is shown during unloading in particular, and it is quite different from what is shown in metals. Such deformation during unlading in polymers has been clarified experimentally [1] and theoretical formulation of the deformation has been attempted [2].
The authors have employed a viscoplastic constitutive equation which was formulated by Murakami et al. [3] by combining the kinematic hardening creep theory of Malinin and Khadjinsky [4] with the nonlinear kinematic hardening rule of Armstrong and Frederick [5]. Then the authors have attempted to modify the nonlinear kinematic hardening rule in order to describe the strain recovery of polymers during unloading so far.
In the present study, a loading surface will be defined in a viscoplastic strain space, and a criterion of loading-unloading will be defined by using the loading surface. Moreover, a parameter will be defined by using the loading surface, and the nonlinear kinematic hardening rule will be modified by using the parameter in order to describe the peculiar deformation of polymers during unloading.

VISCOPLASTIC CONSTITUTIVE EQUATION

A Small deformation is assumed for simplification here. The total strain ε_{ij} is assumed to be the sum of an elastic strain ε_{ij}^{e} and a viscoplastic strain ε_{ij}^{vp} : i.e., $\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{vp}$. The elastic deformation is subjected to the Hooke's law.
The viscoplastic strain rate $\dot{\varepsilon}_{ij}^{vp}$, on the other hand, is derived from the kinematic hardening creep theory of Malinin and Khadjinsky [4]. The viscoplastic deformation is assumed to be incompressibility and independence on the hydrostatic stress, and a viscoplastic potential g is given by an effective stress $\xi_{ij} = s_{ij} - a_{ij}$ as $g = (1/2)\xi_{ij}\xi_{ij}$ where s_{ij} is a deviator of the stress σ_{ij} and a_{ij} is that of a back stress α_{ij} which is the center of the viscoplastic potential surface in the deviatoric stress space. Then the viscoplastic strain rate $\dot{\varepsilon}_{ij}^{vp}$ is derived as $\dot{\varepsilon}_{ij}^{vp} = \Lambda(\partial g / \partial \sigma_{ij}) = \Lambda \xi_{ij}$ where Λ is a positive scalar function depending on a stress and a stress path.
An equivalent effective stress $\bar{\xi}$ and an equivalent viscoplastic strain rate $\dot{\bar{\varepsilon}}^{vp}$ are defined as $\bar{\xi} = [(3/2)\xi_{ij}\xi_{ij}]^{1/2}$ and $\dot{\bar{\varepsilon}}^{vp} = [(2/3)\dot{\varepsilon}_{ij}^{vp}\dot{\varepsilon}_{ij}^{vp}]^{1/2}$. Then the viscoplastic strain rate $\dot{\varepsilon}_{ij}^{vp}$ is derived as $\dot{\varepsilon}_{ij}^{vp} = (3/2)(\dot{\bar{\varepsilon}}^{vp} / \bar{\xi})\xi_{ij}$. In this equation, the relation between the equivalent effective stress $\bar{\xi}$ and the equivalent viscoplastic strain rate $\dot{\bar{\varepsilon}}^{vp}$ is necessary. In the present study, the Sorderberg's rule, $\dot{\bar{\varepsilon}}^{vp} = n[\exp\left(\bar{\xi} / K\right) - 1]$, is adopted where n and K are material constants.
Finally, the back stress α_{ij} is represented by the sum of a linear term $\alpha_{ij}^{(1)}$ and a nonlinear term $\alpha_{ij}^{(2)}$: i.e., $\dot{\alpha}_{ij} = \dot{\alpha}_{ij}^{(1)} + \dot{\alpha}_{ij}^{(2)}$. The linear term $\alpha_{ij}^{(1)}$ is given by the Prager's rule as $\dot{\alpha}_{ij}^{(1)} = A\dot{\varepsilon}_{ij}^{vp}$ and the nonlinear term $\alpha_{ij}^{(2)}$ is given by the nonlinear kinematic hardening rule of Armstrong and Frederick [5] as $\dot{\alpha}_{ij}^{(2)} = b(C\dot{\varepsilon}_{ij}^{vp} - \alpha_{ij}^{(2)}\dot{\bar{\varepsilon}}^{vp})$ (AF model) where A, b and C are material constants.

MODIFICATION OF EVOLUTION EQUATION OF BACK STRESS

A loading surface $f = 0$ is defined in a viscoplastic strain space as $f = (2/3)\varepsilon_{ij}^{vp}\varepsilon_{ij}^{vp} - \rho^2$ where ρ represents a radius of the loading surface $f = 0$ which is depicted as a hypersphere. Development of the loading surface and a criterion of loading-unloading are prescribed as follows:

$$\dot{\rho} = \begin{cases} \dot{\bar{\varepsilon}}^{vp} & f = 0 \text{ and } n_{ij}\dot{\varepsilon}_{ij}^{vp} \geq 0 \quad \text{loading} \\ 0 & f < 0 \text{ or } n_{ij}\dot{\varepsilon}_{ij}^{vp} < 0 \quad \text{unloading} \end{cases}, \qquad n_{ij} = \frac{\partial f / \partial \varepsilon_{ij}^{vp}}{\left[\left(\partial f / \partial \varepsilon_{mn}^{vp} \right) \left(\partial f / \partial \varepsilon_{mn}^{vp} \right) \right]^{1/2}},$$

where $\dot{\bar{\varepsilon}}^{vp}$ is a rate of change in the equivalent viscoplastic strain $\bar{\varepsilon}^{vp}$ with respect to time t.

Then a parameter Ψ is introduced [3]. The parameter Ψ is defined by the relation between the loading surface and the present state of a viscoplastic strain as $\Psi = [\rho - \bar{\varepsilon}^{vp} \operatorname{sgn}(\dot{\bar{\varepsilon}}^{vp})]/2\rho$. The parameter Ψ is always 0 during loading. On the other hand, the parameter Ψ has some value depending on the relation between the loading surface and the present state of a viscoplastic strain only during unloading.

In the present study, the nonlinear kinematic hardening rule of Armstrong and Frederick is modified by using the parameter Ψ in order to describe significant strain recovery during unloading. Following two models are discussed.

Model 1: $\dot{\alpha}_{ij}^{(2)} = b(C_1\dot{\varepsilon}_{ij}^{vp} - C_2\Psi^m\dot{\varepsilon}_{ij}^{vp} - \alpha_{ij}^{(2)}\dot{\bar{\varepsilon}}^{vp})$, Model 2: $\dot{\alpha}_{ij}^{(2)} = b(1-\lambda\Psi)(C\dot{\varepsilon}_{ij}^{vp} - \alpha_{ij}^{(2)}\dot{\bar{\varepsilon}}^{vp})$,

where C_1, C_2, m and λ are material constants.

In the Model 1, the transient recovery term [3] is introduced into the AF model, and the term works only during unloading. In the Model 2, on the other hand, a rate of evolution in the nonlinear term of a back stress changes depending on the value of the parameter Ψ during unloading.

RESULTS OF SIMULATION

Experimental results for polyethylene [6, 7] are simulated by using the viscoplastic constitutive equation and two modified models where the Model 1 or the Model 2 is used instead of the AF model in the constitutive equation.

Figure 1 shows stress-strain relations under monotonic compression at constant total strain rates. Figure 2, on the other hand, shows creep curves under constant compressive stresses. In the figures, each symbol represents the experimental result [6] and each line is a result of the simulation. There is no difference among three models, and viscoplastic deformation of polyethylene can be described by the viscoplastic constitutive equation proposed in the present study.

Figure 3 shows a stress-strain relation in shear deformation at a constant strain rate in which the loading direction is reversed at γ =0.13. Significant strain recovery appears immediately after the reverse of the direction, and the AF model can not describe the behavior adequately. The modified models, on the other hand, can describe the strain recovery, and prediction by the Model 2 is closer to the experimental result [7] than that by the Model 1.

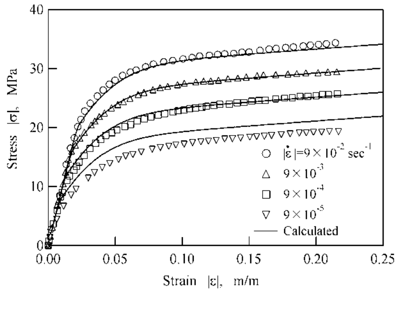

Fig.1 Monotonic compression.

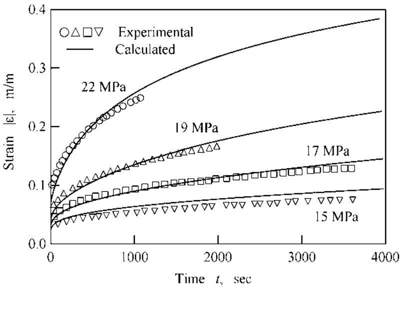

Fig.2 Creep curves.

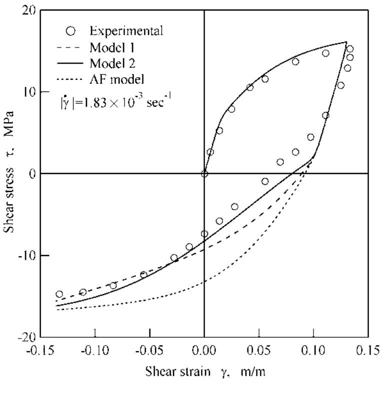

Fig.3 Strain recovery during unloading.

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