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SELECTED PROBLEMS OF DISCRETE-CONTINUOUS MECHANICAL SYSTEMS WITH LOCAL NONLINEARITIES

Amalia Pielorz

Kielce University of Technology, Faculty of Management and Computer Modelling, Al. 1000-lecia Panstwa Polskiego 7, PL 25-314 Kielce, Poland

Summary In the paper nonlinear discrete-continuous systems torsionally, longitudinally and transversally deformed with local nonlinearities having softening characteristics are considered. Two sets of functions are proposed for the description of nonlinearities. In the discussion a wave approach leading to solving differential equations with a retarded argument is applied. Detailed considerations and numerical results are presented for a multi-mass torsional system and for a single gear transmission.

INTRODUCTION

The considerations concentrate on the dynamic analysis of nonlinear discrete-continuous models of various elements and machines. Such models consist of rigid bodies connected by means of elastic elements. To these models also belong those where the motion of elastic elements can be described by the classical wave equation. The use of this equation gives some limitations for the group of the systems under considerations; on the other hand, it enables to apply the solution of the d'Alembert type leading to equations with a retarded argument. Such an approach allows to consider the systems torsionally, longitudinally or transversally deformed. Some examples of such systems one can find in [1-4].
In the discrete-continuous systems local nonlinearities can be incorporated. The variety of nonlinear problems is wide. It may include weak and strong nonlinearities, hardening and softening characteristics. The presence of local nonlinearities in the discrete-continuous models of mechanical systems is justified by engineering solutions and it can have important consequences for their overall dynamic behaviour.
In the paper it is assumed that local nonlinearities have softening characteristics and they are described by two sets of nonlinear functions.
The external loading can be described by an arbitrary function, periodic or nonperiodic. Damping is taken into account by means of an equivalent internal and external damping.

NONLINEAR FORCES AND GOVERNING EQUATIONS

Consider the nonlinear discrete-continuous systems consisting of an arbitrary number of elastic elements connected by rigid bodies. The cross-sections of the elastic elements are constant and they can be torsionally, longitudinally or transversally deformed. In the considered discrete-continuous models a single local nonlinearity by means of a nonlinear spring is taken into account and it represents mechanical properties of various elements having a nonlinear characteristics. It is assumed that these characteristics are of a soft type.
The forces in the nonlinear spring can be described by various nonlinear functions. In the case of hard characteristics the polynomial of the third degree satisfactory describes such forces, as it is shown in [3] for a discrete-continuous torsional system. In the case of nonlinearities having softening characteristics, two sets of nonlinear functions are proposed in the dynamic analysis of appropriate discrete-continuous systems.
The first set of nonlinear functions consists of the polynomial of the third degree with negative coefficients standing by the nonlinear term, and of the sinusoidal function, the hyperbolic tangent function and the exponential function. The parameters in these functions are chosen in such a way that all the functions give the same linear case, the polynomial function is the extension of the sinusoidal function and all functions have similar maximum values. The proposed functions are assumed in the following form

$$1)\ F(X) = K_1 X + K_3 X^3 ,$$

$$2)\ F(X) = A\sin(BX) , \qquad 3)\ F(X) = A\tanh(BX) ,$$

$$4)\ F(X) = A(-1+\exp(BX)) \ \text{ for } \ X \le 0 \ \text{ and } \ F(X) = A(1-\exp(-BX)) \ \text{ for } \ X \ge 0 ,$$

and the parameters of the above functions satisfy the relations $AB = K_1$, $AB^3 = -6K_3$, where X is a displacement connected with the location of the local nonlinearity.
The second set of nonlinear functions consists of the irrational functions, the hyperbolic tangent function and the exponential function. The irrational functions are assumed in the following form

$$5)\ F(X) = K_1 X - K_w(-X)^w \ \text{ for } \ X \le 0 \ \text{ and } \ F(X) = K_1 X + K_w X^w \ \text{ for } \ X \ge 0 ,$$

and the relations $AB = K_1$, $A = K_1 X_m + K_w (X_m)^w$, $X_m = (-K_1 / wK_w)^{1/(w-1)}$ are satisfied now by the parameters of the proposed functions. We are interested in the characteristics of a soft type, so the coefficient standing by the nonlinear term has negative values, and according to the satisfied relations all three functions give the same linear case and have the same maximum values.

The determination of the displacements in the cross-sections of the elastic elements in the considered discrete-continuous systems is reduced to solving several classical wave equations with appropriate initial and nonlinear boundary conditions. Seeking the wave solutions for equations of motion and substituting them into the boundary conditions, the nonlinear differential equations with a retarded argument are obtained. For each discrete-continuous model these equations have a slightly different form, however generally they can be solved numerically by means of the Runge-Kutta method.

NUMERICAL RESULTS

In numerical calculations the external loading as a harmonic function is assumed and the solution is sought in steady states. The detailed results are obtained for two discrete-continuous systems: for a multi-mass torsional system and for a single gear transmission. It is found that the polynomial function, the sinusoidal function and the irrational functions have some restrictions for their application. Namely, for certain parameters describing the considered systems the escape phenomena can be observed. That means that there exist intervals of the frequency of the external loading for which the solutions diverge to infinity. Some such cases are shown in Fig. 1 for amplitudes P_A of nonlinear forces in a single gear transmission when using irrational functions with several values of the exponent w.

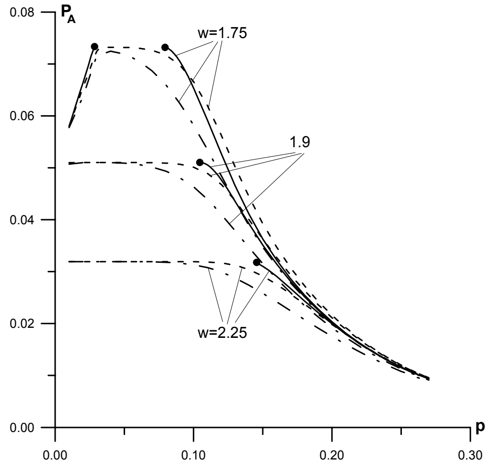

Fig. 1. Amplitude-frequency curves for nonlinear forces in a single gear transmission.

CONCLUSIONS

In the paper it is shown that local nonlinearities can be incorporated in the dynamic analysis of discrete-continuous systems being torsionally, longitudinally or transversally deformed. Though various nonlinear functions can be used for the description of these nonlinearity, some of them have restrictions for their application. It concerns the polynomial function, the sinusoidal function and the irrational functions. The application ranges indicate when the above functions have to be replaced by the remaining nonlinear functions. Two sets of functions are proposed for the description of the local nonlinearity. The first set is suggested for strong nonlinearity and the second one for weaker local nonlinearities.
In the case of local nonlinearities having a hardening characteristic the polynomial function of the third degree can be used. In this case no restrictions were noticed, [3].

References

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