MODELLING BALLAST BEHAVIOUR USING A THREE-DIMENSIONAL POLYHEDRAL DISCRETE ELEMENT METHOD.


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Summary The deterioration of the railway track under a large number of train runnings, in particular the settlement mechanism of the ballast layer or the lateral buckling of track, remains insufficiently known. In order to study the behaviour of ballast layer a three-dimensional Discrete Element Method (DEM), based on the Non Smooth Contact Dynamic (NSCD) approach has been developed.

INTRODUCTION

Since the beginning of the railways, ballasted tracks have been widely employed because of their flexibility from the point of view of the construction and the maintenance but also, because they fulfill well the mechanical requirements imposed by the transport of heavy loads at long distance. The ballast is one of the elements that support the track. The ballast bed consists of a layer of coarse grained material and ensures several functions such as the transmission of the static and dynamic loads to the railway platform and the transversal and longitudinal anchoring of the track.

The track modelling deals with various time and size scale ranges. Ballast is often modelled as a continuous medium. Such a modelling should be relevant when considering the whole track system but this is not necessary the case when studying a small part of the ballast, since the thickness of the ballast layer is only about ten grains under the sleepers. We developed an approach, at the grain scale, resulting from research on the granular medium, referred to as discrete element method.

The three dimensional approach is a new model developed in order to introduce a more realistic physical model. It is much more difficult to conceive an experimental set up to validate, with a sufficient accuracy, the three-dimensional approach but some instructive applications like lateral resistance of the track can be performed.

THREE-DIMENSIONAL DISCRET ELEMENT APPROACH

Discrete element method (DEM) present an interesting alternative to the continuous model or discrete/continuous approach [5] for a better understanding of mechanisms occurring in railway track. It makes possible to model geometrical discontinuities of material and to take into account the grains rearrangements. The Research and Technology Department of the SNCF, supported by RFF 1, undertook, in collaboration with the LMGC, the development of a discrete element software for ballast modelling.

Several computational techniques are currently available in the simulation of collections of rigid bodies. The most popular ones, commonly called Distinct Element or Discrete Element Methods, derive from the work of P.A. Cundall [2]. They consist in approximating the mechanical constraint of non inter-penetrability of each body pair by some close-range steep repulsion law.

A different approach is used in this lecture, characterized by the absence of smoothing approximation. This approach is called (Non Smooth) Contact Dynamics Method [3]. For each rigid body we can write the motion equation with Newton-Euler equation :

\[
M \ddot{\mathbf{x}} = P(t) + r \\
I \dot{\omega} + \omega \times (I \omega) = M_P(t) + M_r
\]

where \( \mathbf{x} \) denotes the position of the center of mass, \( \omega \) the angular velocity, \( P(t) \) and \( M_P(t) \) the resultant and the moment of the external forces, \( r \) and \( M_r \) the resultant and the moment of the contact efforts, \( M \) the mass matrix and \( I \) the inertia matrix. The second equation of this system is possibly nonlinear because of the second term of the left hand side expression.

The “contact” contribution \( r \) comes from interaction between bodies. These laws are written in term of relative algebraic distance \( \Phi(q) \) and velocity \( U(q) \) between the two bodies. If we consider a contact \( \alpha \), it comes from classical kinematic analysis that \( U^\alpha = H^\alpha(q) \dot{q}(q) \), and from duality considerations (conservation of the power expressed with either local variables or global variables) that the contribution \( r^\alpha \) due to the local reaction \( R^\alpha \) satisfy the relation \( r^\alpha = H^\alpha(q) R^\alpha \).

To identify these interactions we need geometrical information about the contact, an algorithm to investigate geometrical intersection between convex polyhedrons has been developed [4]. In fact \( r \) is the sum of all these contributions. The mapping \( H^\alpha(q) \) are linear and transposed one from the other.

We use the local form which require a condensation of the global dynamical equations in term of local unknown variables (relative velocities).

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Considering a contact $\alpha$, we can rewrite the equation (1) in the following manner with $nc$ the total number of “contacts”:

$$U^\alpha = U^\alpha_{free} + W^{\alpha\alpha} h R^\alpha + \sum_{\beta=1, \beta \neq \alpha}^{nc} W^{\alpha\beta} h R^\beta$$  \hspace{1cm} (2)

We can observe that if we “freeze” contact contribution for $\beta \neq \alpha$ this equation express a linear relation between $U^\alpha$ and $R^\alpha$. The solution $(U^\alpha, R^\alpha)$ which verify this equation, velocity Signorini condition and the Coulomb’s friction law can be obtain with an iterative Newton solver [1]. Then the global solution is very simple, we will solve these couples of equations (dynamical and interaction) for each contact considering the other one’s as “frozen”. We will consider that the method has converged once the solution for each contact does not change. This is something like a nonlinear block Gauss-Seidel algorithm.

**LATERAL RESISTANCE EXPERIMENTS**

The evaluation of the influence of vehicle induced forces on the lateral stability of the track has been a major research concern for several decades. The load capacity of the track or its "lateral strength" to handle these loads is a key requirement for track alignment retention, hence to safe train operation. The growing tendency toward higher speeds and heavier axle loads tends to make the issue of track lateral stability more critical.

A numerical experiment to study lateral resistance implies the preparation of a sample representative of a portion of railway track. We decided to take into account the ballast granulometry (25-50 mm) and a sleeper of the type ‘VAX U41’ usually used on high speed tracks. The sample composed of 25000 digitized ballast grains is prepared by deposition under gravity (fig. 1).

![Figure 1. A numerical sample.](image1)

![Figure 2. Results from numerical experiment.](image2)

On this kind of sample it is possible to undertake specific studies for example to analyse the influence of the shape of the sleeper on the efforts distribution or the influence of the granulometry of the ballast on lateral resistance. Initially we studied the influence of friction coefficient between the grains and the sleeper on lateral resistance. We applied an incremental lateral force on the sleeper with different friction coefficients, and we record its displacement. The curves force-displacement (fig. 2) show that the friction coefficient has not a very significant role on the threshold of release of the phenomenon of shifting but has an influence on the amplitude of displacement following the shifting.

**CONCLUSION**

The model proposed is an alternative to the finite elements methods to study ballast behaviour. In spite of the numerical difficulties, contact detection between polyhedron, local problem of contact and friction, the model allows us to study a large collection of rigid bodies with a polyhedral shape. The use of discrete element method for a better understanding of the buckling phenomenon of track show the accuracy of this approach to simulate a real behaviour of ballast. It is possible to study micromechanics quantities not easily measurable by experiments like stress or influence of friction coefficient.

**References**


