HERTZ CONTACT AT FINITE FRICTION AND ARBITRARY PROFILES.

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Extended summary.

A consistent and robust method is described to solve normal axisymmetric contact problems at smooth and convex but otherwise arbitrary profiles. Special emphasis is put on the presence of finite friction causing partial slip between dissimilar solids. It is shown that at partial slip the evolving relative stick-slip contour is independent of any convex contact profile at monotonic loading. For flat and conical profiles with rounded edges and apes, relations between force, depth and contact contours are given together with surface stress distributions. For dissimilar solids, full field values were computed individually. The location and magnitude of critical stress measures are determined for a range of geometrical and material parameters.

The theory laid down by Hertz (1882) for normal frictionless contact between two nonconforming bodies of elliptical profiles stands as a landmark in linear elasticity. It would take almost a century until the corresponding problem was attacked for adhesive contact by Mossakovskii (1963) and Spence (1968) and for finite friction by Spence (1975a, 1975b). Already in the Hertz formulation based on linear kinematics, the problem is essentially nonlinear as a moving boundary is present. A further nonlinearity will evolve when finite friction prevails as stick-slip boundaries have to be determined when partial slip occurs. Substantial progress was made by Spence (1975a, 1975b), who showed that under monotonic loading a single stick-slip contour will appear being independent relatively to the contact profile provided it has a polynomial shape. More recently, c.f. e.g. Ciavarella and Hills (1999), Ciavarella (1999), contact of various non-standard profiles such as blunted cones and flat indenters with rounded edges, has been investigated. Besides more general contact law behaviour, the main intention has been to predict initiation of plastic flow or the occurrence of fracture and fatigue. The present effort follows in this spirit.

Mossakovskii (1963) seems to be the first to propose that normal contact problem at adhesive behavior may be attacked in two steps, by first solving the problem at an incremental advance and subsequently apply superposition. Emphasizing self-similarity for power law profiles, further progress was made by Spence (1968, 1975a, 1975b). Mossakovskii and Spence were mainly concerned with determination of surface tractions and displacements at contact. In case of axisymmetric and frictionless contact it was later shown by Hill and Storåkers (1990) that complete field values may readily be determined by a solution for incremental fields followed by cumulative superposition along radial paths. Numerically such a procedure is at advantage as only a stationary mesh is required when finite elements are to be used. The strategy was applied in full by Storåkers and Larsson (1994) for Norton creep by combining a finite element procedure with cumulative superposition, which for the case of linear viscosity corresponds to linear elasticity. In the present linear elastic case history dependence evolves through the presence of finite friction. It will be shown though that this issue is only fictitious as when partial slip arises, the stick-slip contour relative the external contact contour will be constant. This prevails for any contact profile provided that is smooth and convex and the loading is axisymmetric and monotonically increasing.

The problem to be analysed involves mutual impression of two isotropic elastic solids at normal contact. It is assumed that the two solids are homogeneous and have axisymmetric and convex though otherwise arbitrary smooth surface profiles. For simplicity and clarity, the problem is first posed for a rigid punch indented into an elastic half-space later to be generalized for the case of two dissimilar elastic solids.

Two axisymmetric punch shapes were investigated in particular, flat indenters with rounded corners and conical ones with rounded tips. The two profiles are described by

$$f(r) = \begin{cases} 0 \\ (r-b)^2/2R \end{cases}, and \qquad f(r) = \begin{cases} r^2/2R & r \le b \\ (2br-b^2)/2R & b \le r \le a \end{cases} \qquad (1)$$

in obvious notation.

When finite Coulomb friction applies, μ, following Spence (1975a) for power law profiles, $f \sim r^p$, it is assumed *a priory* that only an interior stick region, $0 \le r \le c$, and an exterior slip region, $c \le r \le a$, evolve. It will be shown that this holds true for any profile, *f(r),* which is smooth and convex.

The problem so formulated involves a moving contact boundary, *a*, and a stick-slip contour, *c*, which have to be determined as part of the analysis. In a general situation when the contact profile, *f(r)*, is arbitrary the problem has to be treated incrementally and accordingly formulated in rate form.

Introducing reduced variables

$$a\tilde{x}_i = x_i, \qquad \dot{h}\tilde{u}_i(\tilde{x}_k) = \dot{u}_i(x_k, a) \qquad (2)$$

where *h(a)* is the indentation depth, it may first be observed that at an incremental change, the reduced problem corresponds formally to that of flat punch of radius unity indented to a unit depth with an unknown relative stick-slip radius, c/a.

A finite element procedure, based upon the commercial code ABAQUS (2002), was developed to solve the intermediate flat die problem in reduced variables. The method adopted was essentially based on an earlier analysis of normal frictionless indentation as explained in detail by Storåkers et al. (1997) with the influence of finite friction subsequently taken into account by Carlsson et al. (2000). It should be observed in particular that the moving boundary is reduced to a stationary one.

Once the reduced problem has been solved, field values of the original problem may be found by simple cumulative superposition through integration along radial paths as proposed by Hill and Storåkers (1990). By quadrature of eq. (2), displacement fields follow as

$$u_i(x_k, a) = \int_0^a \tilde{u}_i(\tilde{r}) h'(s) ds \tag{3}$$

and stress fields alike.

The missing link between the physical depth of indentation, *h(a),* and contact profile, *f(r)*, is given by a Volterra integral equation

$$h(r) - \int_0^r \tilde{u}_z\left(\frac{r}{s}\right)\frac{dh}{ds} ds = f(r) \tag{4}$$

which is readily solved by standard methods.

The problem discussed so far has involved only contact between a rigid indenter and an elastic half-space. Accordingly, the interior fields to be determined, as by a finite element method, require solutions only for a half-space. When two dissimilar elastic solids are in contact, in general, solutions for interior fields are required in a full space. By a linear transformation, it has been shown explicitly by Mossakovskii (1963) and Spence (1968,1975a,1975b), that the resulting contact tractions may be directly determined by aid of a half-space solution for a tailored combination of material parameters. As a result, two uncoupled problems pertinent to dissimilar half-spaces with prescribed normal and tangential stresses are obtained and may be readily solved individually by the ABAQUS (2002) procedure.

Some selected results are illustrated for particular situations. The invariant relative stick-slip contour is shown in detail as a function of the friction coefficient, μ, and Poisson's ratio, ν, or alternatively the Dundurs parameter. For rounded conical and flat profiles, besides global contact laws, normal and tangential surface stresses are shown as functions of profile geometry, *b/a* cf. eq. (1), and μ. The location of maximum tensile stress is determined in order to predict Mode I fracture initiation. Interior fields are used to determine peak values of von Mises stress to predict initiation of plastic flow. The dissimilar case is analysed for different surface profiles, compliance values and friction coefficients. In particular it is shown where and when fracture and plastic flow is expected to occur.

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