A CREEP CONTINUUM DAMAGE THEORY FOR BEAMS, PLATES AND SHELLS

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Summary The widely used approach in modelling the creep-damage behavior of structures is the continuum damage mechanics, which is based on constitutive equations for the creep strain rate tensor, and evolution equations for internal state variables. This approach is usually not compatible with those theories of beams, plates and shells which are based on cross-section approximations of displacement and/or stress fields. In this case there is no unique possibility to transform three-dimensional constitutive and evolution equations to the averaged equations for tensors of forces and moments and the corresponding strain or strain rate measures. We discuss an extension of the theory of "simple shells" recently proposed for problems of finite elasticity to the creep-damage analysis. The balance equations are formulated directly without any cross-section approximation. A general structure of creep-damage constitutive equations is discussed considering the symmetries of a shell and within the framework of continuum thermodynamics. Based on simplified examples of rods and plates we compare our approach with the three-dimensional analysis by the finite element method.

In many applications thin-walled structures (beams, plates, pipes, pipe bends, etc.) operate at elevated temperatures. Under such conditions the behavior of metals and alloys is primarily determined by irreversible time-dependent creep and material deterioration processes. In order to estimate the long-term behavior it is important to understand the mechanisms of the time-dependent stress redistribution and damage growth, particularly in the zones of nozzles, pipe connections and welds. The widely used approach in modelling the creep-damage behavior is the continuum damage mechanics, which proposes constitutive equations for the creep strain rate tensor, evolution equations for damage variables, and states nonlinear initial-boundary value problems in order to perform the structural analysis. With the progress in the material science and continuum mechanics, many new constitutive models have been developed which include physically motivated state variables and which are able to consider different effects in material behavior such as the damage induced anisotropy, stress state dependence of damage evolution, etc. These models can be incorporated into a commercial finite-element code in order to analyze time-dependent deformations and stresses in a thin-walled structure under specific mechanical loading. One way is the "three-dimensional approach" which is based on balance equations of the three-dimensional continuum. This approach seems more preferable for creep analysis since the existing constitutive models of creep are developed with respect to the Cauchy stress and strain (rate) tensors, and the proposed measures of damage (scalars or tensors of different rank) are defined in the three-dimensional space. Another way is the use of the structural mechanics of beams, plates and shells, and the balance equations formulated in terms of force and moment tensors. This approach often find application because of simplicity of model creation, smaller effort in solving non-linear initial-boundary value problems of creep, and easily interpretable results. In [1, 2] numerical solutions of the cross-section assumption based equations for shells and the three dimensional equations of creep-damage mechanics are presented. It is found that any theory of beams, plates or shells derived as an approximate version of the three-dimensional equations requires "more accurate" cross-section approximations for the transverse normal and shear stresses if damage processes are taken into account. The type and the order of such approximations cannot be established a priori. Furthermore, there is no unique way to formulate the creep-damage constitutive equations for assumed stress resultants and the corresponding strain measures. In this sense it is more reliable to solve the three-dimensional equations which are "free" from ad hoc assumptions for the displacements and stresses. However, in this case the question about the relevance of "three-dimensional" creep-damage constitutive assumptions for the analysis of thin-walled structural elements is still open.

The aim of this presentation is to recall the governing equations of creep mechanics, to derive a continuum damage shell theory and to compare the long-term predictions based on the standard three-dimensional approaches and the proposed two-dimensional shell model. First, we summarize the basic features of creep behavior in polycrystalline metals and alloys and thin-walled structures. Second, we derive the governing equations of creep damage mechanics based on the model of ”simple shell” [3]. Within this model a shell is assumed to be a two-dimensional continuum in which the neighboring parts interact by means of forces and moments. Based on this assumption the balance equations are formulated by the direct approach without incorporating any cross-section approximation for displacement and/or stress fields. The constitutive equations are formulated with respect to the tensors of forces and moments and to the shell type strain measures, i.e. the membrane strains, changes of curvature and transverse shear strains. The general structure of the rate equations for the description of the creep process is established within the framework of the theory of symmetry and applying the energy balance equation as well as the second law of thermodynamics. Furthermore, we introduce the damage measures suitable for the analysis of thin-walled structures. The tertiary creep behavior is described by the use of the effective stress concept and the strain rate equivalence principle. As a special case we discuss a class of constitutive equations linearized with respect to the strains. The proposed model takes into account the stress state dependence of damage evolution, e.g. the different damage rates in tension and compression.

Another approach of analysis of shells is based on the traditional ”three-dimensional” continuum damage mechanics. In this case the shell must be treated as a three-dimensional continuum. We introduce the widely used Kachanov-Rabotnov-Leckie-Hayhurst creep-damage model [4] and demonstrate the description possibilities for stress states which are typical for thin-walled structures.

In order to compare the approaches, we introduce the examples of a transversely loaded beam and of a transversely loaded plate. The material constants are taken for the type 316 stainless steel at 650°C from [5]. The ”three-dimensional” solution is performed by ANSYS finite element code by the use of solid type elements. For the comparison a standard ”quasi three-dimensional” solution by the use of shell type finite elements is performed. In this case the cross section approximations are applied. The averaged equilibrium conditions formulated for stress resultants are solved step by step. At each time step the creep and damage rates are calculated from three-dimensional stress fields, which are recovered by means of the assumed cross-section approximations. We demonstrate that these two approaches lead in general to quite different long-term predictions. The agreement between the shell and solid type finite element solutions is obtained only in the case that the damage evolution is controlled by the von Mises stress. However, in this case the tertiary creep rates are the same under tension and compression, which in general, contradicts to the known experimental data for steels.

Finally, we present the results obtained by use of the proposed shell theory. From the results follows, that by suitable formulation of creep-damage constitutive and evolution equations for the shell the best agreement of our theory with the three-dimensional solution can be obtained.

References

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