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3-D STRUCTURAL ACOUSTICS MODELING WITH HP-ADAPTIVE FINITE ELEMENTS

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Summary The Centre has developed a 3-D *hp*-adaptive finite-element structural acoustics code for modeling acoustic scattering from underwater elastic structures. 3-D continuum mechanics is used throughout the computational domain; thin structural components, such as plates and shells, are modeled with 3-D physics rather than plate or shell theories. The paper describes an unusual approach to code development, explains the underlying physics and mathematics, and presents several scattering and propagation models.

INTRODUCTION

Man-made undersea structures are typically enclosed thin shells (e.g., submarines, mines) with interior reinforcements (e.g., rib-stiffeners, partitions). For over thirty years the structural acoustics community has been developing finite-element codes to model the radiation and scattering of sound from such structures. Despite all this effort, there are still large discrepancies in the acoustic fields computed by different codes, for even the simplest structures at low frequencies, e.g., differences of 5 to 10 dB or more (well over 100%) for a simple closed shell with minimal interior structure at $ka \cong 2$. It has been the hypothesis of the first author for several years that these very large differences are probably due to the underlying physics, i.e., the use of plate and shell theories, which are 2-D physics inside a 3-D geometry. Near structural discontinuities, such as shell intersections, the elastic field is strongly 3-D, with complicated evanescent elastic waves in the vicinity of the discontinuities. Errors in these localized fields will produce errors in the propagating elastic waves, which in turn will produce errors in the acoustic waves “launched” into the exterior fluid. In short, the scattered or radiated acoustic field may depend strongly on the localized 3-D elastic fields in the shell.

Motivated by this hypothesis, the Centre initiated a research effort five years ago to develop a high-fidelity 3-D finite-element structural acoustics code for modeling acoustic scattering from underwater elastic structures, primarily undersea mines located in or near the seabed in shallow water. Thin structural components, such as plates and shells, are modeled with 3-D physics rather than plate or shell theories. The code is therefore fully 3-D, i.e., in both physics and geometry. The first version of a steady-state (time-harmonic) code, named FESTA, for Finite Element Structural Acoustics, was released in 2003.

CODE DEVELOPMENT APPROACH

The Centre is using the commercial software package ProPHLEX [1] developed by the R&D Division of Altair Engineering (formerly COMCO). This package is a suite of software tools for *developing* customized FE codes. It contains a library of several thousand routines [O(10^6) lines] that perform all the detailed tasks of a FE analysis. To develop an application code, the researcher derives the Galerkin residual equations from the governing PDEs and then links self-written code [O(10^3) lines] with the vendor-written library, which creates an application code with a commercial-quality GUI, e.g., FESTA. A central feature of ProPHLEX is its *hp*-adaptivity – an automated procedure for achieving near-optimal FE meshes, based on the use of high-order “hierarchic” basis functions and element-by-element error estimators.

ProPHLEX is applicable to any system or process that can be mathematically formulated as a system of coupled 2nd-order PDEs, in the following tensor form:

$$\left(a_{\alpha l\beta m} q_{\beta,m}\right)_{,l} + b_{\alpha\beta m} q_{\beta,m} + c_{\alpha\beta} q_{\beta} = -f_{\alpha} \tag{1}$$

where $(\ )_{,i} = \partial(\ )/\partial x_i$ and x_i are the independent variables. For physically oriented systems, x_i are usually spatial coordinates. The components of the vector q_β may be any meaningful quantities, but q_β need not be sensible as a vector. For example, the components can include displacements in an elastic solid and pressures in an acoustic fluid. The coefficients $a_{\alpha l\beta m}$, $b_{\alpha\beta m}$ and $c_{\alpha\beta}$ may be a function of q_β (for nonlinear processes) and/or a function of the independent variables x_i (for spatial inhomogeneities) and/or a function of time (for transient processes). All variables must be real-valued; hence, any PDEs with complex variables must be decomplexified. Equation (1) can describe a very broad range of phenomena, including the coupling of different types of phenomena.

ProPHLEX uses eq. (1) in each and every finite element in the computational domain. Therefore, to develop a structural acoustics code, the analyst must cast the governing PDEs for both the vibrating solid and acoustic fluid into a single system of PDEs in the form of eq. (1). This, in effect, treats the entire fluid-structure domain as a single continuum, which is modeled using only one type of finite element: a fluid-solid element.

A UNIFIED CONTINUUM MECHANICS DERIVATION OF A FLUID-SOLID ELEMENT

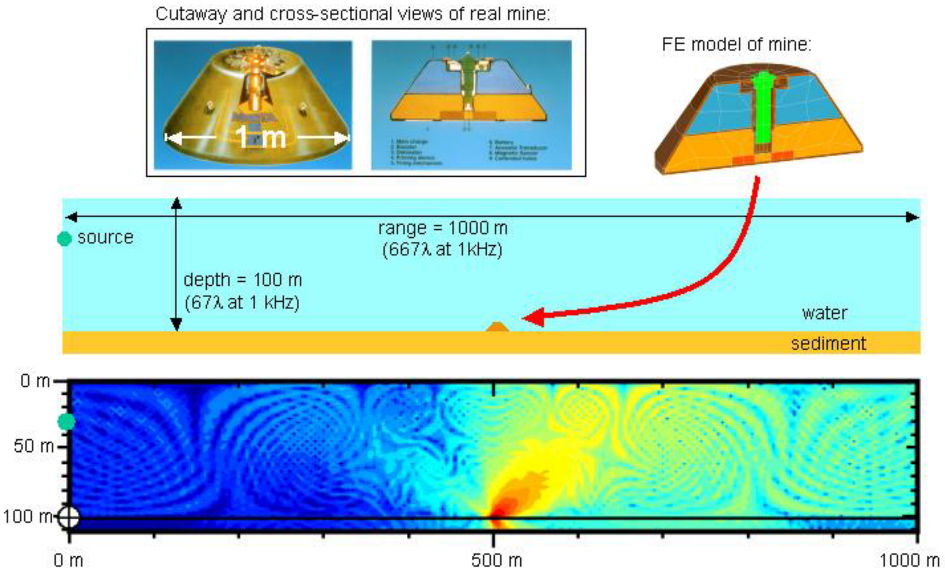
Both the solid and the fluid are elastic media, experiencing small, steady-state vibrations about a quiescent state. The solid may be viscoelastic, anisotropic and inhomogeneous. The fluid is inviscid but may include volume dissipation, and is isotropic and inhomogeneous. Under these conditions, *the continuum mechanics for the fluid are a special case of those for the solid*. Hence, to derive the acoustic wave equation for the fluid, one may begin with the governing PDEs for wave propagation in an elastic solid and apply the appropriate limiting conditions. The wave equations in both media can then be combined into a single tensor wave equation in the form of eq. (1), where

$$q_\beta = \left\{\begin{array}{c} u_x^R \\ u_y^R \\ u_z^R \\ u_x^I \\ u_y^I \\ u_z^I \\ p^R \\ p^I \end{array}\right\} \begin{array}{l} \uparrow \\ \text{displacement} \\ \text{in solid} \\ \downarrow \\ \uparrow \\ \text{acoustic pressure} \\ \text{in fluid} \\ \downarrow \end{array}, \quad a_{\alpha l\beta m} = \begin{array}{c|ccc} & \beta=1\;2\;3 & 4\;5\;6 & 7\;8 \\ \hline \begin{array}{c}\alpha=1\\2\\3\end{array} & c^R_{\alpha l\beta m} & -c^I_{\alpha l(\beta-3)m} & 0 \\ \begin{array}{c}4\\5\\6\end{array} & c^I_{(\alpha-3)l\beta m} & c^R_{(\alpha-3)l(\beta-3)m} & 0 \\ \begin{array}{c}7\\8\end{array} & 0 & 0 & \dfrac{1}{\omega^2\rho^f}\delta_{\alpha\beta}\delta_{lm} \end{array}, \quad c_{\alpha\beta} = \begin{array}{c|ccc} & \beta=1\;2\;3 & 4\;5\;6 & 7\;8 \\ \hline \begin{array}{c}\alpha=1\\2\\3\end{array} & \omega^2\rho^s\delta_{\alpha\beta} & 0 & 0 \\ \begin{array}{c}4\\5\\6\end{array} & 0 & \omega^2\rho^s\delta_{\alpha\beta} & 0 \\ \begin{array}{c}7\\8\end{array} & 0 & 0 & K_{\alpha\beta} \end{array} \tag{2}$$

The superscripts *R* and *I* indicate real and imaginary parts, respectively. Equation (2) shows the α,β-“plane” of the 4th-order tensor $a_{\alpha l\beta m}$; each of the 64 positions is a 3×3 tensor in l,m, where $l,m = 1,2,3$ for the 3 spatial coordinates. The 4th-order tensors $c^R_{\alpha l\beta m}$ and $c^I_{\alpha l\beta m}$ are the real and imaginary parts, respectively, of the complex elastic moduli. The fluid and solid densities are ρ^f and ρ^s, respectively, and $K_{\alpha\beta}$ is the decomplexified fluid compressibility.

EXAMPLE PROBLEM

The oral presentation will show several models of acoustic propagation and target scattering, including verification vis-à-vis analytic solutions and other numerical codes. One of the most challenging of these problems is shown here: scattering from a realistic mine sitting on a sedimentary seabed in shallow water (see figure below). The ocean surface and water/sediment interface are flat. The sediment is modeled as a heavy, dissipative fluid. A 1 kHz monopole source is located at a range of 500m from the mine and a depth of 30m. Although FESTA can model scattering in complex waveguides (as will be shown in the oral presentation), this computational domain is too large in terms of wavelengths, λ, for available computer resources. Since λ = 1.5m, the source/mine separation is 333λ and the depth is 67λ. A high-accuracy 3-D scattering model would require several tens of millions of DOF. Therefore a more efficient *hybrid* approach is employed: FESTA computes the local scattered field in the vicinity of the mine and a non-FE propagation code computes the large-scale propagation of the incident and scattered fields in the waveguide. The propagation code used in this problem is OASES [2], which is based on wavenumber integration. A 3-step method couples the inputs and outputs of the two codes. The final scattered field throughout the waveguide is shown by the contour plot.



CONCLUSIONS

Using *hp*-adaptive finite-element technology, a 3-D structural acoustics code has been developed for modeling acoustic scattering from underwater elastic structures. 3-D continuum mechanics is used in both the fluid and solid media, making the code 3-D in both physics and geometry. The code continues to be verified against a variety of test problems.

References

[1] Liszka, T.J., et. al.: ProPHLEX – An *hp*-adaptive finite element kernel for solving coupled systems of partial differential equations in computational mechanics. *Comput. Methods Appl. Mech. Engrg*. **150**: 251-271, 1997.

[2] Schmidt, H.: OASES, Version 3.1, User Guide and Reference Manual. Dept. of Ocean Engrg., M.I.T., (2003). Available at http://acoustics.mit.edu/faculty/henrik/oases.html