PROBLEMS OF APPLICATION OF HIERARCHICAL MODELLING, DISPLACEMENT FEM AND A POSTERIORI RESIDUAL ERROR ESTIMATION TO STATIC AND DYNAMIC ADAPTIVE ANALYSIS OF COMPLEX STRUCTURES

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Summary The paper addresses the main problems of implementation of the idea of hierarchical modelling in the scope of displacement finite element methods and of application of the a posteriori residual error estimation to adaptive analysis of complex structures. These structures consist of solid, thin- and thick-walled parts and transition zones as well. Our presentation is limited to linear static and linear dynamic (modal) analyses. The main source of the problems under consideration is incompatibility of the various mechanical models forming hierarchy of models for complex structures, locking, improper solution limit and boundary effect sensitivity of the displacement finite element formulation, and similar sensitivity of the a posteriori error estimation by the Element Residual Methods. The elaborated methods of overcoming the mentioned problems are elucidated, if available. For those of the problems which have not been resolved yet, the suggested methods are indicated and their potential for overcoming the problems are thoroughly discussed.

GENERAL OVERVIEW OF COMPLEX STRUCTURE ANALYSIS

The main difficulty in complex structures analysis by FEM is that conventionally each part of the structure, i.e. solid, thin- and thick-walled part is described with the different mechanical model. Such models may correspond to various theories, for example: first order shell, higher order shell, hierarchical shell or three-dimensional elasticity one. It is known that for each of the listed theories different field unknowns are used. As a consequence also different degrees of freedom of the corresponding finite element methods are utilised. For this reason analysis of complex structures is usually difficult. That is also why, in comparison to enormous number of works devoted to analysis of solids, shells and plates, the papers concerning all them together as one complex structure are really rare.

Some achievements in adaptive finite element methods for solids and shells [1] as well as recent advances in hierarchical modelling of the first order shells [2] or hierarchical shell models [3, 4] has become an impulse for the authors of this work to extend these potentially promising and powerful ideas into the complex structure description and analysis. The results of the research on hierarchical modelling and adaptive finite element methods for complex structures static analysis has been recently presented in [5]. The idea is now being extended to linear dynamic (modal) problems. The idea utilises hierarchical modelling applied to first order shell, hierarchical shell and three-dimensional elasticity, as well as two-dimensional, three-dimensional or mixed *hp* approximation spaces, which both lead to local (element level) model, *h*-, *p*- and *q*-adaptive finite element methods based on displacement formulation. The numerical parameters *h*, *p* and *q* represent averaged dimension of the element, as well as longitudinal and transverse orders of approximation, respectively. The two-dimensional or three-dimensional isotropic or anisotropic adaptive procedures are controlled through a posteriori total, modelling and approximation error estimation based on the Residual Equilibration Method. The main feature of the proposed approach is overcoming of the problem of different degrees of freedom in various parts of the complex structure by means of application of the three-dimensional approach which consists in utilisation of three-dimensional degrees of freedom only, regardless mechanical models applied. Note that in contrast to other researchers' approach, we managed with our method to build one hierarchy of models including various mechanical models instead of exclusion of the models which do not fit the main hierarchy of the predecessors', i.e. which are not compatible with either hierarchical shells originated from higher order theories or with the first order shell hierarchy or with hierarchy of numerical models conforming three-dimensional elasticity.

For the sake of honesty we should mention also attempts of the other researchers, which could be utilised in analysis of complex structures. These works address only one aspect of adaptive complex structure analysis, i.e. the modelling error issues and are quite rare [6, 7]. The recent contribution of Stein and Ruter [8] suggest many practical hints for proper adaptive modelling within complex structures but seems to propose different methods that could lead to effective complex structure analysis. In this paper we will try to show that some of the problems listed in [8] has already been resolved in [5] partly or completely or can be potentially resolved within our approach.

MAIN PROBLEMS, THEIR SOURCE, AND OVERCOMING THEM

Apart from the main implementation problem of hierarchical modelling of complex structures resulting from different degrees of freedom of different theories used for description of various parts of complex structures there are also other implementation problems. They deal with application to complex structures of the most common displacement formulation of the finite element methods, and with employment of the Residual Equilibration Methods (REM) for error estimation. The displacement FEM is sensitive (in the sense that special approach is necessary) to such phenomena as: the improper solution limit of the 3D elasticity model when transverse order of approximation q equals one, shear or shear-membrane lockings, and boundary layers typical for thin- or thick-walled members of the complex structures. Also REM may suffer from the latter two numerical phenomena. It is also sensitive to edge and corner singularities.

Hierarchical modelling

Even though the proposed 3D approach enables usage of the same three-dimensional degrees of freedom within all the complex structure, regardless of the locally applied models, this approach does not remove incompatibility of the first order Reissner-Mindlin model with the hierarchic shell models and with the three-dimensional elasticity. The source of this incompatibility are: the plain stress assumption and kinematic assumption of the theory (no elongation of the normals to the mid-shell surface during deformation). The incompatibility due to the second of the mentioned assumptions can be removed through application of the transition approximations of the displacement field between the first order shell model and hierarchic shell models or three-dimensional elasticity model, while the first one is an inherent feature of the first order shell models and cannot be easily resolved. In our proposition we are not afraid of the latter incompatibility. The method proposed for such situations is controlling the modelling error within the transition zone. This can be done with the same methodology which is used for the first order model itself. Such a control leads to movement of the transition area into the region where this error is acceptable. If such an action is not effective enough then the first order model has to be replaced with the first of the higher order shell models compatible with the description of the remaining part of the complex structure.

Displacement finite element methods

Three phenomena – improper solution limit of the 3D elasticity model (with transverse order of approximation q equal to one), locking, and boundary layers need special treatment within displacement FEM formulation (this is also true for some of these phenomena within other FEM formulations). The approach proposed in this work offers local (element level) a posteriori numerical tools for detection and assessment of the mentioned phenomena. To the best of the authors' knowledge the idea is totally original and has been proposed and implemented first in [5]. For the improper solution limit phenomenon we propose detection numerical tool, for the locking phenomena (either shear or shear-membrane) we present tools for their detection and assessment of their strength, while for boundary layers we have to detect and assess strength and range of the edge effect. The additional advantage is that the proposed tools are compatible with the error estimation procedures that facilitates their implementation. In our proposition two local problems have to be solved for the chosen element of the potentially affected region and the obtained solutions have to compared. Note that with these tools at hand we try to move adaptive solution for the complex structure static or dynamic problems towards the asymptotic convergence region, where adaptivity controlled with error estimation can be performed.

A posteriori error estimation

In contrast to a typical case of simple structures a posteriori error estimation within complex structures has to include not only the approximation error but also the modelling error which results from application of mechanical models simplified with respect to three-dimensional (3D) elasticity. For the purpose of approximation and modelling error estimation we propose application of the Residual Equilibration Method for all mechanical models included, i.e. 3D-based first order and hierarchic shell models and 3D elasticity model as well. The method is applied twice. First we apply it to approximation error estimation for the elements conforming three mentioned models. Note that the global approximation error being a sum of element error indicators possesses upper bound property for these models. For the second time the method is applied for total error estimation within the elements conforming two shell models. The upper bound property of the global total error estimate can be proved for the hierarchic shell models only. Local modelling error indicators are obtained as a difference of the total and approximation indicators.
Some improvement of the estimation in the case of locking, boundary effect and boundary singularities can be gained by constraining local problems or introduction of higher order splitting functions for equilibration instead of linear ones.

CONCLUSIONS

The elaborated general method constitutes an interesting proposition towards obtainment of an effective numerical tool for linear static or dynamic adaptive analysis of complex structures. Some problems await resolution or improvement, however.

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