ELLIPTICAL INSTABILITY IN A ROTATING SPHEROID

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Summary
This study concerns the elliptical instability of a flow in a rotating deformed sphere. The aim of our work is to observe and measure the characterics of this instability in experiments and to compare them with theorical predictions. For this purpose, an elastic and transparent hollow sphere has been moulded. The flow is visualised using Kalliroscope flakes as the sphere is set into rotation and compressed by two rollers. The elliptical instability occurs by the appearance of the so-called 'spin-over' mode whose growth rates and saturations are measured for different Eckman numbers by video image analysis. These growth rates compare avantageously to theorical calculations which are performed using classical asymptotic expansions. The linear analysis is then completed by a non linear model which predicts correctly the asymptotic regimes for high Eckman numbers. Some results that concern the elliptic instability in a rotating deformed spherical shell or the triangular instability will also be presented.

INTRODUCTION

The elliptical instability is a classical three-dimensionnal instability of fluid dynamics. It is known to affect and even to destroy vortical flows. As described in the review by Kerswell [1], the instability mechanism lies on a resonance phenomenon between inertial (Kelvin) waves and external strain fields. Therefore, this instability appears to be very attractive to explain the three-dimensionnal undulations of non axisymmetric vortices distorted by strain fields [2] as it is the case in turbulence [3] or vortex dynamics [4, 5]. But another major interest of the elliptical instability concerns the behavior of the flows contained in rotating ellipsoids which are laboratory models of astrophysical objects subjected to tidal distorsions of close gravitationnal bodies [6, 7, 8]. More specifically, the occurence of this instability in the motion of the molten iron cores of planets, such as the Earth, would be of prime importance in the understanding of the generation and the dynamics of the magnetic fields of planets [7]. With this aim, Aldridge et al [9] have performed computations and built a rotating deformable shell where they observed some indications of the presence of the elliptical instability in the spherical geometry. Using the technique invented by Malkus [10], and more recently intensively used by Eloy *et al.* [11], we have been able to apply an elliptical constraint to a deformable rotating sphere and measure the characteristics of the flow instabilities by image analysis and finally compared them to the analytical treatment of the instability [12].

EXPERIMENTAL APPARATUS

The hollow sphere is moulded in a cylindrical block of transparent silicone gel. The cylinder is then placed in the device used by Eloy *et al.* [11]. It is set in rotation versus its axis while it is gently compressed by two rollers parallel to the rotation axis. A video camera records the motions of water seeded by kalliroscope particles and enlighted by a laser sheet. A snapshot of the fluid flow is shown on figure 1a.

STABILITY ANALYSIS

The stability of a fluid in a rotating sphere which is slightly elliptically deformed is analysed. It is shown that two linear neutral modes associated with the rotation of the fluid can resonate with the strain induced by the deformation. The condition of resonance can be expressed in term of azimuthal wavenumber and frequency. This resonance leads to a temporal growth of the instability mode amplitude. The so-called spin-over mode is shown to be always the most unstable mode in the range of Ekman number E and deformation rate ε of the experimental flow. This mode is a solid body rotating flow whose rotation axis is perpendicular to the main rotation. The growth rate of this unstable mode can be analytically calculated and a weakly non linear model is developed to predict an observed saturated regime for peculiar values of E.

RESULTS

Figure 1a shows a typical visualisation of the unstable spin-over mode in a meridional plane. The typical S shape of the rotation axis is due to the combination of the main rotation, the spin-over mode and viscous boundary layers. A video analysis permits to measure the tilting angle of the rotation axis as a function of time. Then an estimation of the growth rate can be evaluate and compare to the theoretical prediction (see figure 2).
On figure 1b, one of the experimental video analysis is compared to the weakly non linear model. The model is in agreement with the saturation observed and measured in the experiment. This regime is observed for sufficiently large value of E. If E is smaller than a critical value, small structures destroy the spin-over mode and then more complex dynamic is visualized. For a peculiar value of E, we also observe an intermitent regime between spin-over and peripherical small structures.

a) b)

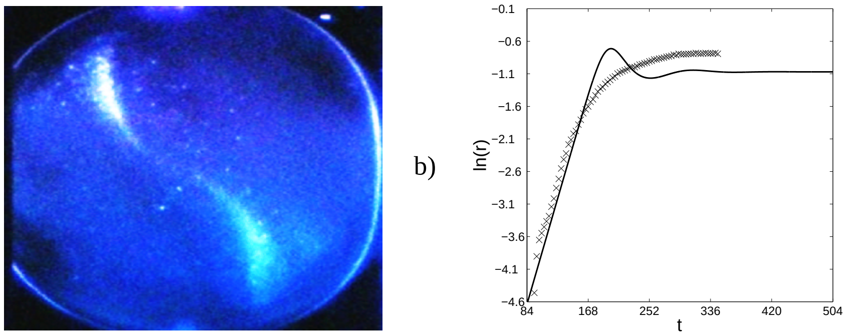

Figure 1. a) Visualisation of the spin-over mode in a meridional plane. b) Comparison between the experimental results ($\times$) and the nonlinear model. ($E = 2.5\ 10^{-4}$ and $\varepsilon = 0.16$).

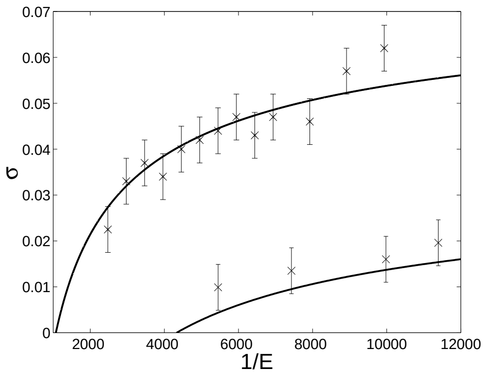

Figure 2. Growth rate versus the inverse of the Ekman number for $\varepsilon = 0.08$ (lower) and $\varepsilon = 0.16$ (upper). Experimental results ($\times$) and linear theory (solid line).

The flow dynamic in a deformed spherical shell has also been investigated. The linear growth rate is in this case measured with laser Doppler anemometry. Comparisons with the linear stability theory have also been carried out for this configuration.

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