RELAXATION TIME FOR SEDIMENTING SPHERES OF A SUSPENSION WITH PERIODIC BOUNDARY CONDITIONS

Maria L. Ekiel-Jeżewska*, Eligiusz Wajnryb*
*Institute of Fundamental Technological Research, Polish Academy of Sciences, Świetokrzyska 21, PL 00-049 Warsaw, Poland

Summary We numerically simulate sedimentation of a suspension with periodic boundary conditions. We consider mono-disperse non-Brownian spheres in low-Reynolds-number fluid flow. We analyze how fast a steady state is reached and how the evolution depends on the initial pair distribution.

INTRODUCTION

Sedimentation of suspensions made of spherical mono-disperse non-Brownian particles has been recently extensively investigated experimentally, theoretically and numerically [1]. The main interest has been to investigate the nature of the steady state and to understand how does it differ from equilibrium. Explanation of the observed large correlations of particle velocity fluctuations and theoretical derivation of the relevant pair probability distribution at the steady state are still open problems.

It is of interest to understand how fast and in what way sedimenting suspensions become stationary. The observed relaxation times are long, both in experiments [3, 4] and simulations [5, 6]. Some authors claim that in some cases a steady state is not reached at all [2]. Experiments of Guazzelli et al. [3, 4] indicate that there exist two characteristic time scales in evolution of a sedimenting suspension. First, after several Stokes times only, a big swirl is formed in the container. Later, after a few hundreds of Stokes times, the system becomes stationary. The problem how the relaxation time depends on the size of the system has been raised and investigated [2, 6]. Results of Ref. [7] indicate that time of interaction between sedimenting spheres is sensitive to initial conditions. Therefore in this work, within a toy-model, we investigate qualitatively how does the relaxation time of a sedimenting suspension depend on its initial pair distribution.

INITIAL CONFIGURATIONS – A TOY MODEL

We consider exemplary system with N=8 sedimenting spheres in each periodic cube. We select small volume fraction ϕ=0.003, as in the system investigated experimentally in Refs. [3, 4]. The corresponding size of the periodic cube $L \approx 11.2$. Length is measured in sphere diameters, time and velocity in the Stokes units.

We study the following three classes of initial positions of the sphere centers.

- EQUILIBRIUM – positions are chosen with the most-probable distribution (uniform with no overlap).
- PERTURBED LATTICE (deficit of close pairs) – we perform a random perturbation from cubic lattice with a small amplitude $A = (L-2)/8 \approx 1.1$, with the uniform distribution.
- CLOSE PAIRS (excess of close pairs) – spheres are grouped in pairs, each pair in a separate ball of diameter $D = 4$. We take the most-probable distributions of the spheres within the ball and of the balls within the cube.

The integrated pair distribution $n(r)$ for each class is plotted in Fig. 1. Here $n(r)$ is the mean number of particles in a spherical volume of radius r surrounding a test sphere, minus the average number of particles in the same volume.

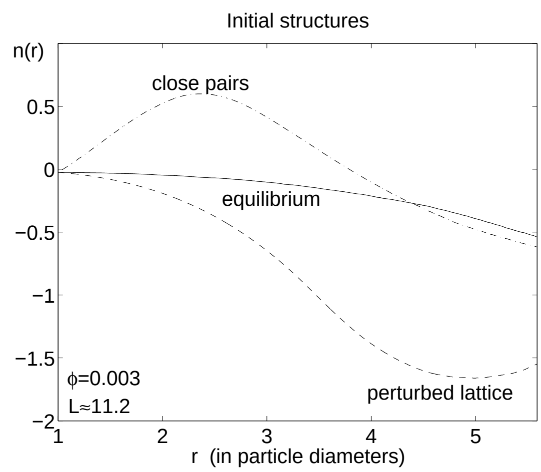

Fig. 1. The initial integrated pair distribution $n(r)$.

METHOD

The fluid is described by the incompressible Stokes equations with periodic boundary conditions and the stick boundary conditions at the surfaces of N spheres immersed in the periodic cube. The Green tensor for such a system has been specified by Hasimoto [8] and expressed in a form suitable for fast numerical computation by Cichocki and Felderhof [9]. Their results are implemented in the numerical codes used in this paper, which are modifications of the previous codes developed for infinite fluids and applied e.g. in Ref. [10]. For given positions of N spheres, the mobility coefficients are evaluated by the multipole method [11], with the lubrication correction [12], which takes care of interactions between close spheres. To get accurate trajectories, we truncate the multipole expansion at the multipole number $l = 3$ (with l defined in Ref. [11]). Equations of motion for N spheres are integrated with the fourth-order Runge-Kutta algorithm. In this way the particle positions and velocities are determined as functions of time.

RESULTS

The averaging is performed over all the particles in the periodic cell and over a number M of random initial configurations, separately within each of three classes listed above. In Figs. 2 and 3, we trace the mean sedimentation velocity U and the mean horizontal and vertical velocity fluctuations, $\delta \mathrm{U}_\perp$ and $\delta \mathrm{U}_{||}$, respectively, as functions of time. Similar analysis is performed for the integrated pair distribution $n(r)$.

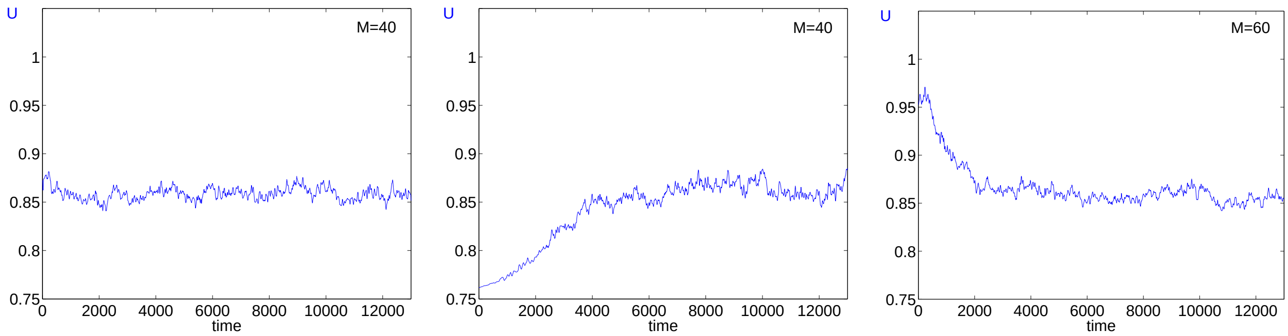

Fig. 2. Evolution of the mean sedimentation velocity U for three classes of initial configurations. Left: equilibrium; center: perturbed lattice; right: close pairs. Time is measured in Stokes times. M is the number of initial configurations.

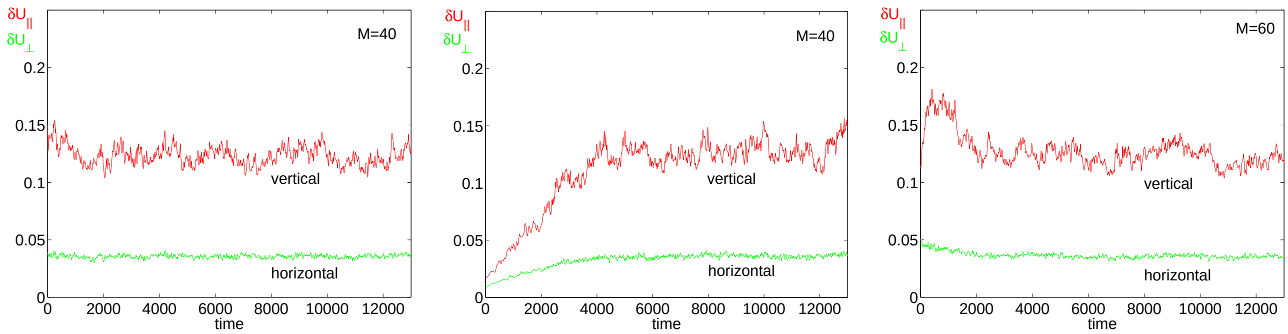

Fig. 3. Evolution of the mean horizontal and vertical velocity fluctuations, $\delta \mathrm{U}_\perp$ and $\delta \mathrm{U}_{||}$, respectively, for three classes of initial configurations. Left: equilibrium; center: perturbed lattice; right: close pairs. Time is measured in Stokes times. M is the number of initial configurations.

Relaxation time to the steady state is of the order of $2000\,\tau_{St}$ for close pairs and $4000\,\tau_{St}$ for perturbed lattice. It increases if the initial deficit (or excess) of particles increases, i.e. if A or D decreases. For initially close pairs, the second much shorter time scale has been observed around $100 - 400\,\tau_{St}$. On this time scale the excess of close pairs, initially concentrated around a given small distance, is redistributed over all the distances. It takes one order of magnitude longer to loose the excess of close pairs, reaching the final steady state.

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