

## DISCRETE MODELS AND THEIR APPLICATION IN DAMAGE MECHANICS

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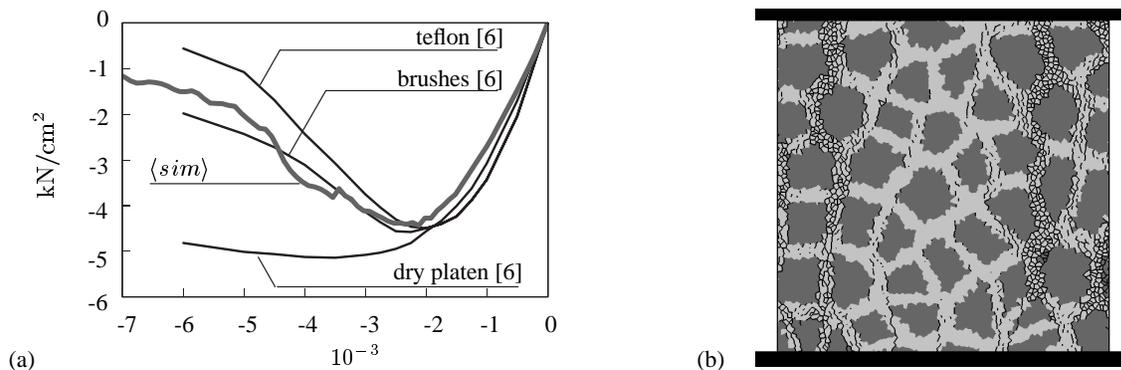
*Summary* We present different discrete element models for the simulation of two-dimensional problems in the context of damage mechanics. Target materials to be simulated are cohesive and non-cohesive geomaterials like concrete- or sand-type materials. The development of adequate homogenization methods supplements the definition of the discrete models.

### DISCRETE ELEMENT MODELS

The failure mechanisms of geomaterials are characterized by complex failure modes and, furthermore, they show a highly anisotropic bias due to their inhomogeneous microstructure. Since localization phenomena like cracks or shear bands occur the material cannot be treated as continuous in the usual manner. If fracture and fragmentation of the solid occurs, the creation and continuous motion of the evolving crack surfaces apparently represent discontinuous phenomena and are difficult to handle. Therefore, most continuum models, and in particular those ones based on continuum damage mechanics, cannot account for the discrete nature of material failure in a natural way and need some extension. Alternatively, discrete particle models like discrete element methods (DEM) have been developed. As the name DEM suggests, a solid is replaced by a discontinuous particle composite which allows for a detachment of bonds between particles (if initially present) and a re-contact of open surfaces. In order to simulate and quantify the full range of geomaterials from non-cohesive ones like sand to cohesive ones like concrete, ceramics or rock, different types of discrete element models are presented. Starting from a basic polygonal two-dimensional DEM model for non-cohesive granular materials, more complex models for cohesive materials are obtained by inclusion of beam or interface elements between corresponding particles. The formulation of the standard DEM model is based on the work in [1] where individual particles are considered as unbreakable and undeformable rigid bodies. The local deformational behavior of the particles is approximated by an elastic repulsive force related to the overlapping area of contacting particles, i. e. the detailed representation of the contact is approximated. As initially presented in [2, 3], cohesive components are included by beams, allowing to model typical features in the failure of geomaterials like concrete or rock in a rather crude way. In order to enhance a quantification of the discrete model instead of beams, interfaces along the common particle edges have been introduced [4, 5, 6]. Thus, the complexity of the model is increased in that the representation of the bonds between particles is more detailed and physically coherent.

### SIMULATION OF A VIRTUAL MICROSTRUCTURE

The last step in the series of increasing complexity is the realization of a microstructure-based simulation environment which utilizes the foregoing enhanced DEM models, confer [4]. In short, the microstructure is included, if different properties of the cohesive components (beam or interface) are assigned with respect to their position, i. e. inside the matrix, inside the aggregate and between aggregate and matrix. With the growing model complexity a wide variety of failure features of geomaterials can be represented, see [3, 4]. Furthermore, the inclusion of a real microstructure which regards for stiffer aggregates embedded in a less stiffer matrix enables a quantification of the model. If the softening of the interface elements (bond between the particles) is monitored, a detailed view of the microstructural softening process is obtained. Seven simulation series of a uniaxial compression test of a quadratic block have been carried out. The simplified picture of the composite structure of one of these simulation series in Fig. 1 (b) highlights the crack propagation through the specimen short after peak load. The dark grey color denotes the aggregate particles and light grey the surrounding matrix. Thereby, either the matrix as well as the aggregates are represented by a finite amount of individual particles. For this reason, the shape of the aggregates is irregular. The black lines in Fig. 1 (b) define those interfaces that have been

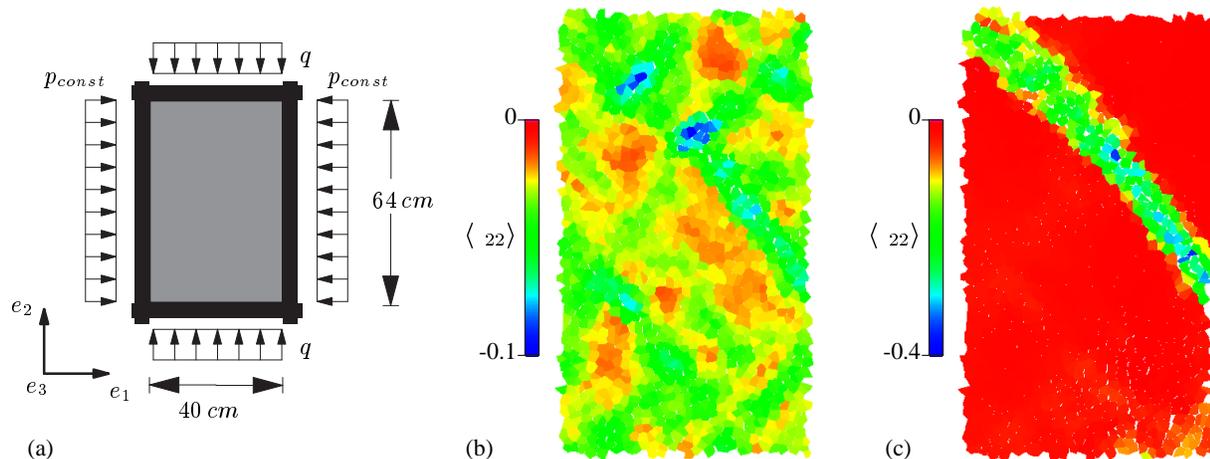


**Figure 1.** (a) Stress-strain diagram of simulations and experiments by [6] and (b) eliminated bonds.

eliminated in the course of the simulation. The average stress-strain curve of seven different (depending on the random generation of composition and position of aggregates) simulation series ( $\langle sim \rangle$ ) is plotted in Fig. 1 (a). The confrontation with experiments on concrete specimens with different boundary conditions [6] expresses the good agreement between simulations and experiments.

## HOMOGENIZATION OF DISCRETE PARTICLE ASSEMBLIES

In parallel, homogenization techniques have been applied which allow us to relate microscopic quantities, like the contact forces and displacements, to corresponding macroscopic quantities, like stresses and strains, confer [4, 7, 8, 9]. Starting point of these homogenization approaches is the argument of scale separation. While the macroscopic problem is defined on a scale with characteristic length  $D$ , a representative or "averaging" volume is of size  $d$ , and, finally, individual particles are of characteristic diameter leading to the scale separation  $D \gg d$ . Utilizing these arguments the balance of linear and angular momentum provide simplified equilibrium conditions for a representative volume element (RVE) on the scale  $d$ . Application of usual averaging approaches on the RVE level yields average stress tensors in terms of contact forces. In a similar manner kinematic quantities like strains and velocity gradients as well as energetic quantities are obtained. The benchmark problem used for the validation of this homogenization procedure is a strain driven biaxial simulation of a cohesionless sample with rigid side walls, compare the setup in Fig. 2 (a) or the references [4, 7, 8, 9]. The target material to be simulated is of sand-type. Therefore, the material structure does not represent a two-phase microstructure as presented in the previous section, i. e. all particles possess the same stiffness and differ only in size. The rectangular sample consists of 2560 particles and is loaded in a strain driven format in vertical direction while a constant side pressure  $p_{const}$  is applied. The evaluation of the average vertical strain distribution  $\langle \epsilon_{22} \rangle$  leads to an interesting observation which is typically obtained in sand- and rock-type materials. The homogenization method provides the opportunity to study the pattern formation in combination with the self-organization of the granular body. Fig. 2 (b) pictures the formation of competing shear zones in the pre-localization regime of the simulation. After reaching the peak only one primary shear zone and reflections of it remain and drive the complete failure of the granular specimen, confer Fig. 2 (c). For clarity, in Figs. 2 (b) and (c) the color scales have been adapted to the maximum strain of the corresponding simulation stage.



**Figure 2.** (a) Biaxial setup and geometry, vertical strains (b) before and (c) after localization.

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