A NONLOCAL PLASTICITY-DAMAGE FORMULATION BASED ON THE MICROMECHANICS OF DEFECT GROWTH

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Summary: A large-strain plasticity formulation is coupled with a nonlocal damage influence in order to model damage development in ductile materials. Two damage variables are used: one which characterises the defect volume fraction and which is predominantly governed by hydrostatic stress and plastic dilatation, and one which describes the average defect shape and is particularly sensitive to shearing. Growth relations for these variables as well as the yield surface are determined from unit cell analyses of defect growth and defect extension.

INTRODUCTION

Industrial forming operations on ductile materials typically result in large plastic strains. Under the influence of these plastic strains, voids and microcracks are initiated, usually at second-phase particles or inclusions. Further straining results in growth and coalescence of the individual defects and may thus lead to a considerable degradation of the overall material strength and of the remaining ductility. In situations where the residual properties after forming are relevant – for instance in view of subsequent forming steps – or where the degradation is used to separate material (e.g. blanking), it is important to quantify the microstructural damage development and its influence on the material behaviour.

Accurate macroscopic models of ductile damage growth can only be developed based on a thorough understanding of the underlying microscopic processes. The formulation of a yield criterion on the basis of unit-cell analyses of void growth by Gurson [1] has been a landmark contribution in this respect. Ductile damage models based on this work are used widely, see for instance [2] for a review. However, although these models provide accurate predictions in cases where relatively high hydrostatic stresses exist, they are much less successful in shear-dominated situations. This limitation is due to the assumption of a spherical void which expands isotropically in deriving the yield criterion. For low triaxialities and large shear strains the defects are stretched and coalesce without much volume increase – an effect that cannot be described by the void volume fraction as used in the Gurson-based models.

A second limitation of existing ductile damage formulations originates in the fact that they are mostly based on local continuum mechanics. This means that when the damage growth leads to an overall softening material behaviour pathological localisation effects are observed and numerical results no longer converge upon refinement of the spatial discretisation. Post-peak results obtained with such formulations should therefore be used with extreme care. The lack of mesh-objectivity is related to the absence in the material model of a material length scale, which can set the width of damage bands and thus regularise the localisation of deformation.

NONLOCAL PLASTICITY-DAMAGE FRAMEWORK

In order to overcome the above limitations of existing ductile damage formulations, a new coupled large-strain plasticity-damage model has been developed, based on earlier work by Geers [3]. The model features two field variables which characterise the damage state of the material. One of them, denoted by f, represents the volume fraction which is occupied by defects. This variable plays a role which is similar to the void volume fraction in the Gurson-based models. An increase of this variable is accompanied by plastic dilatation and occurs mainly as a result of hydrostatic stress. The other damage variable, S, characterises the shape of the defects and can be compared to the aspect ratio used by Gologanu et al. [4]. The evolution of this variable is mainly governed by deviatoric straining, although large volumetric strains may also result in a decrease. As a result, it mainly comes into play in shear-dominated problems.

The influence of damage on the local material strength is reflected in the yield criterion, which depends on the two damage state variables: $\Phi(\boldsymbol{\tau}, \varepsilon_{\mathrm{p}}, f, S) \leq 0$; note in passing that, unlike standard von Mises plasticity, the yield criterion also depends on the hydrostatic part of the stress tensor. Generally speaking, an increase of damage results in a shrinking yield surface and thus in material softening. Care must therefore be taken to formulate the damage growth in such a way that pathological localisation of damage growth and deformation is avoided. For this purpose two additional kinematics-related variables are introduced: a nonlocal effective volumetric plastic strain $\bar{\varepsilon}_{\mathrm{v}}$ and a nonlocal effective deviatoric plastic strain $\bar{\varepsilon}_{\mathrm{d}}$. The evolution of the damage variables is described by evolution laws which related the damage rates $\dot{f}$ and $\dot{S}$ to the nonlocal deformation rates $\dot{\bar{\varepsilon}}_{\mathrm{v}}$ and $\dot{\bar{\varepsilon}}_{\mathrm{d}}$. The nonlocal field variables $\bar{\varepsilon}_{\mathrm{v}}$ and $\bar{\varepsilon}_{\mathrm{d}}$ are related to their local, standard counterparts ε_{v} and ε_{d} respectively by a partial differential equation which was first used in a plasticity context by Engelen et al. [5]. Together with an additional set of boundary conditions these partial differential equations form two boundary value problems which must be discretised and solved simultaneously with equilibrium. The additional differential equations provide spatial interactions at a length scale which appears in the differential operator and which may be related to the average distance between defects. As a result, the localisation process is regularised and damage bands have a positive, finite width.

MICROMECHANICAL BASIS

Establishing the precise structure of the yield function and the evolution laws in the plasticity-damage framework is not trivial. Rather than assuming some phenomenological form for these relations, they are based on micromechanical analyses of defect growth and defect extension. Nucleation of voids or coalescence have not yet been considered. Plane strain finite element analyses have been carried out on unit cells containing elliptic voids of different sizes and shapes in an undamaged elastoplastic matrix. The unit cells have been subjected to a wide range of straining regimes, from purely deviatoric to equibiaxial. In these analyses, the evolution of the void volume fraction f and the shape parameter S, as well as overall stress measures, have been recorded. Examples are given in Figure 1, which shows the shape variable S versus strain as a dotted curve for different initial conditions (left) and different straining directions (right).

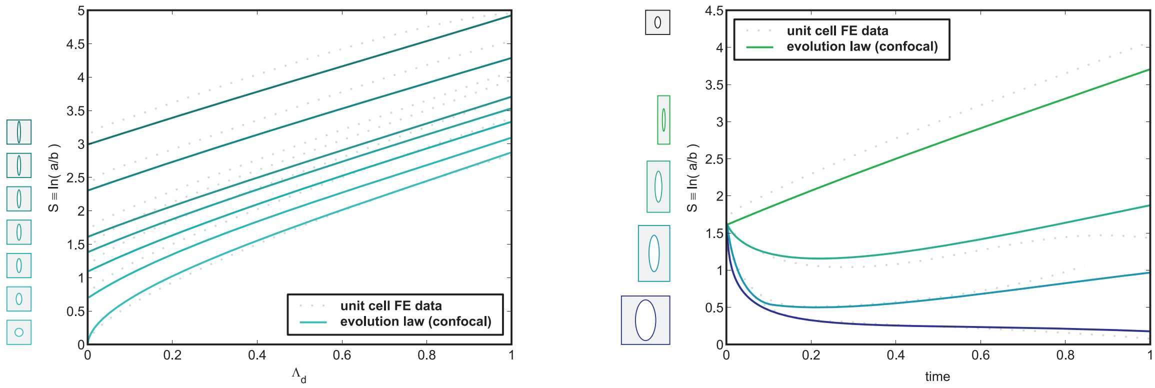

Figure 1: Evolution of the defect shape variables S for several initial shapes (left) and straining directions (right) as computed in the numerical unit cell analyses (dotted curves) and using the fitted evolution laws (solid curves).

Evolution laws for f and S in terms of the overall effective strain rates are formulated based on relatively simple mechanical approximations. In particular, it is assumed that the matrix material is incompressible and that the void and an approximate, elliptic unit cell remain confocal throughout the deformation process. The relations which can be derived from these assumptions pick up all relevant trends observed in the numerical unit-cell analyses. The quantitative agreement is subsequently improved by introducing a few parameters and fitting these to match the numerical results. The resulting fits have been plotted in Figure 1 as solid curves. The structure of the yield function is derived in a similar fashion from an approximation of the plastic work rate together with normality (cf. [1, 4]) and is subsequently adapted to fit the numerical experiments.

CONCLUDING REMARKS

The constitutive relations which result from the analyses as described above exhibit a response which is very similar to that of Gurson's theory in the regime in which this theory is known to perform well, i.e. for relatively high triaxialities. On the other hand, substantially different behaviour is obtained in shear, in which Gurson's theory predicts no damage growth at all. Particularly in the latter regime, however, the response of the model can still be improved by including coalescence effects. It is believed that the micromechanical modelling provides an excellent basis for introducing these effects in a realistic way.

References

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